



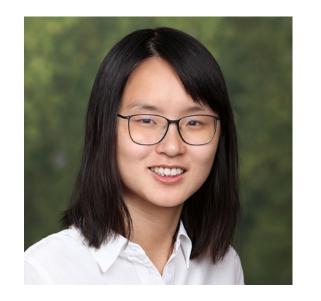
Transformers and Pre-trained Language Models

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Introduction





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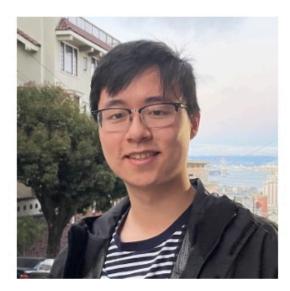
Lucy He (w/ Peter Henderson)



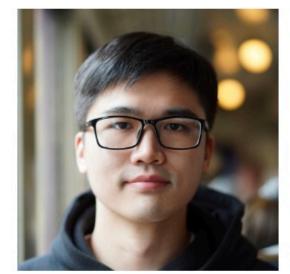
Alexander Wettig



Mengzhou Xia



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Zexuan Zhong

Our research focuses on training, adapting and understanding large language models

See more at https://www.cs.princeton.edu/~danqic/

Lecture plan

Part I. Transformers

Focus: innovations and key designs in neural architectures



Part II. Pre-trained language models

Focus: training objectives & data, downstream adaptations

Lecture plan

- Fundamentals (70%) I will walk through the most important ideas in NLP and LLMs in the past 5+ years (Transformers, pre-training, in-context learning, RLHF, ...)
- How do these ideas evolve and lead to state-of-the-art models? (15%)
 - I will highlight recent improvements and developments
- Cutting-edge research topics (15%) What research topics do we study in 2024? I will briefly discuss some of the works from my research group too

Part I. Transformers

Transformers

Attention Is All You Need

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(Vaswani et al., 2017)



What is attention?

NEURAL MACHINE TRANSLATION BY JOINTLY LEARNING TO ALIGN AND TRANSLATE

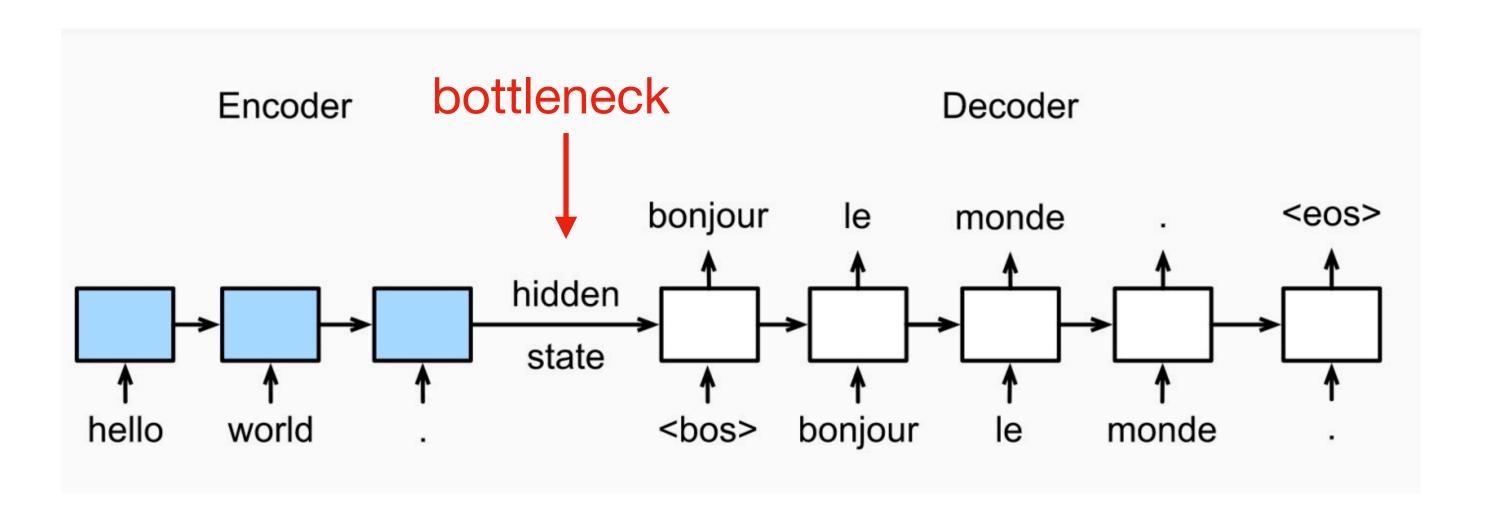
Dzmitry Bahdanau

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KyungHyun Cho Yoshua Bengio* Université de Montréal

(Bahdanau et al., 2015)

 Attention is a technique to address the "bottleneck" issue in the seq2seq model, originally designed for machine translation



What is attention?

NEURAL MACHINE TRANSLATION BY JOINTLY LEARNING TO ALIGN AND TRANSLATE

Dzmitry Bahdanau

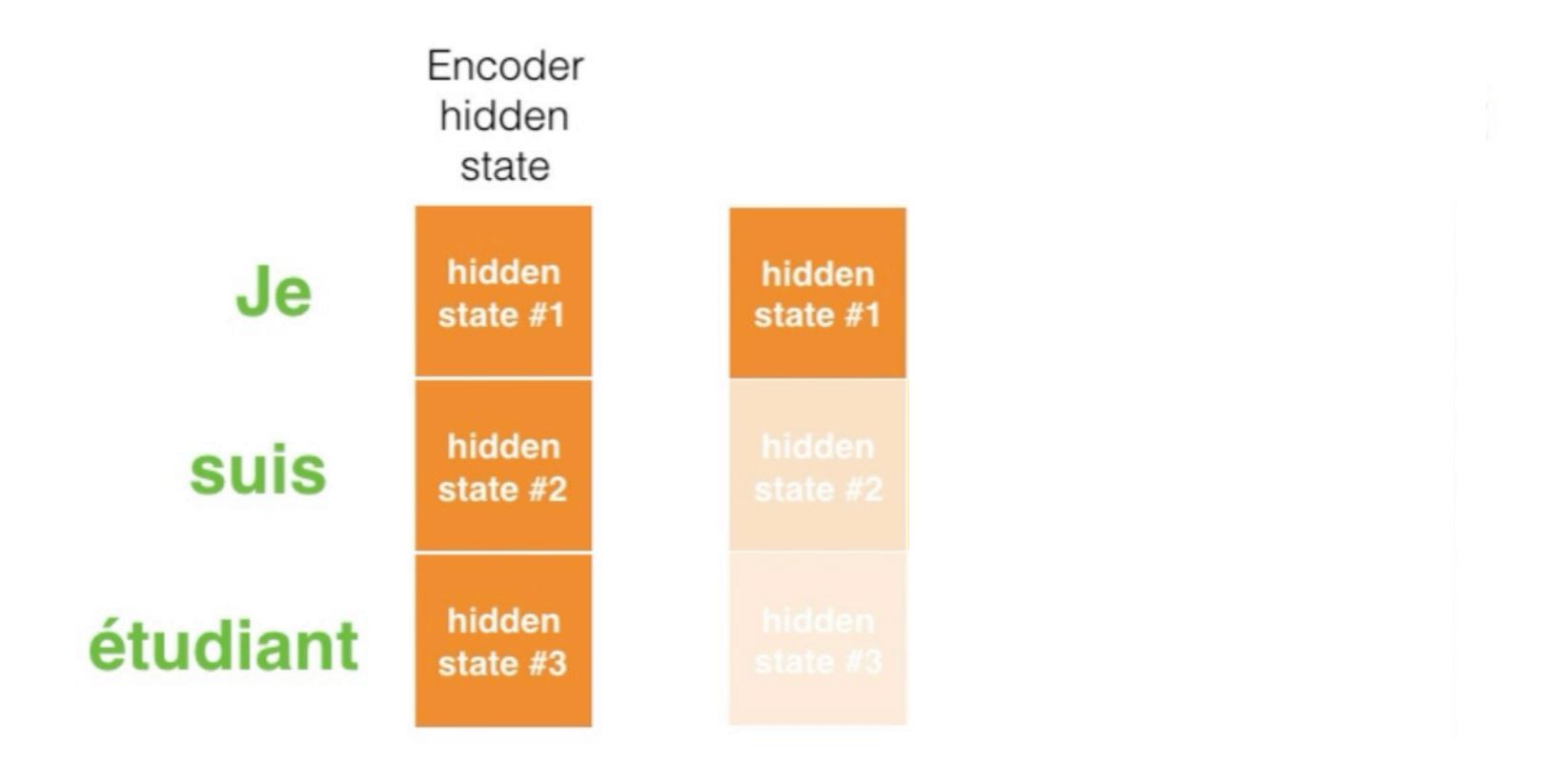
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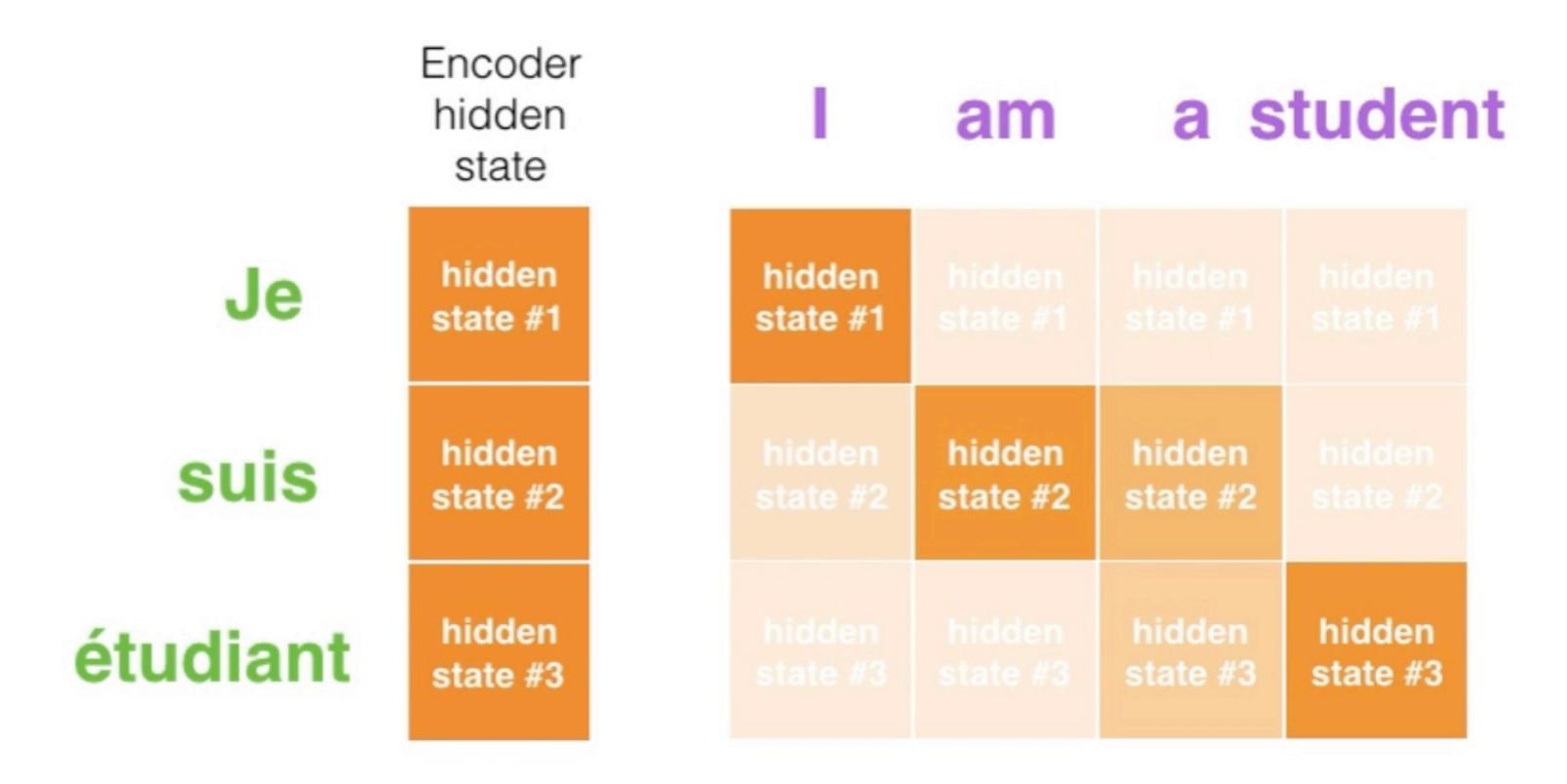
(Bahdanau et al., 2015)

- Attention is a technique to address the "bottleneck" issue in the seq2seq model, originally designed for machine translation
- **Key idea** \mathbb{Q} : At each time step during decoding, focus on only a particular part of source sentence
 - This depends on **decoder's** current hidden state h_t^{dec}
 - Usually implemented as a probability distribution over the hidden states of the encoder ($h_i^{\it enc}$)

Attention for seq2seq models



Attention for seq2seq models

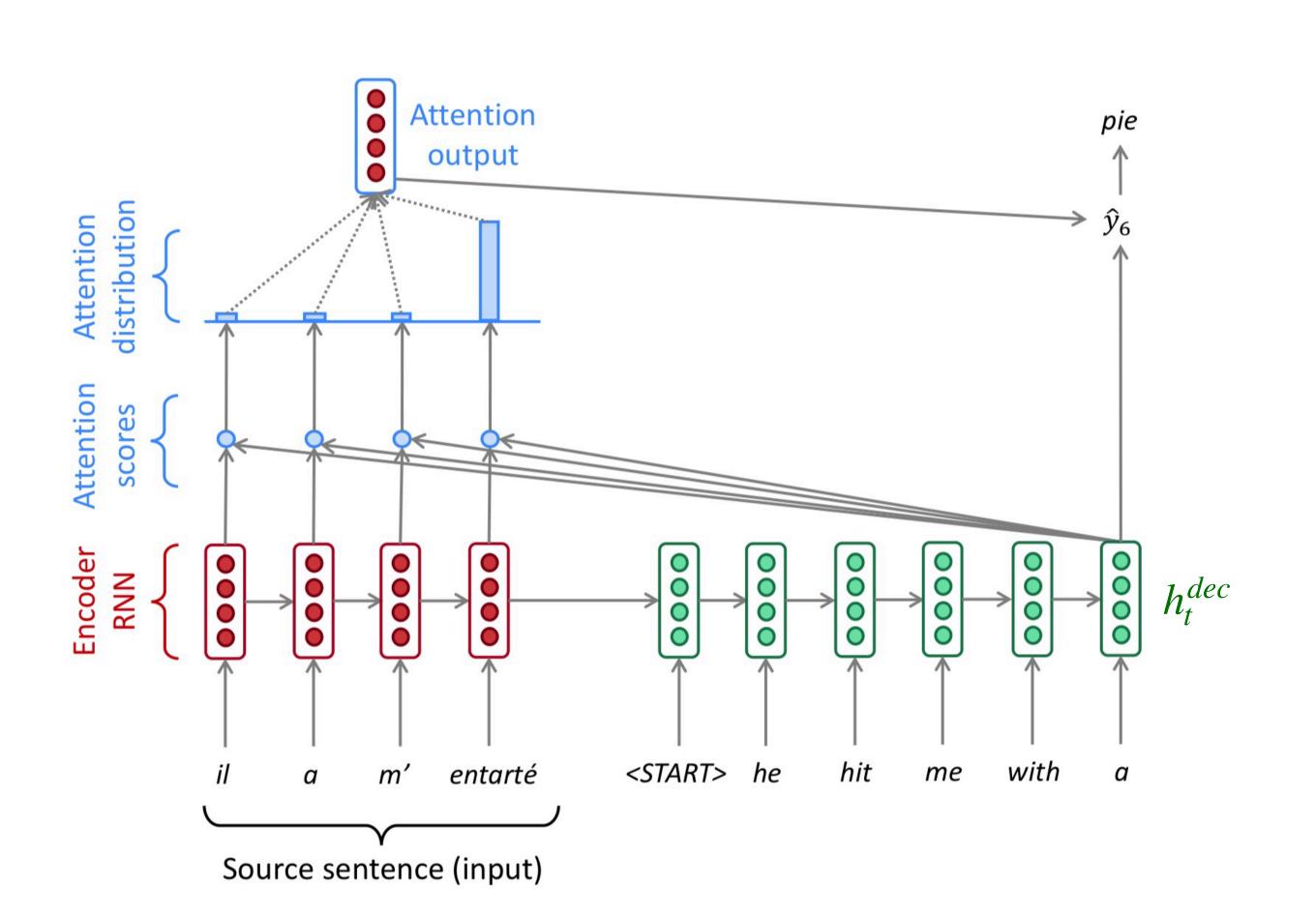


Attention learns the notion of alignment

"Which source words are more relevant to the current target word?"

Attention for seq2seq models

 $h_1^{enc}, \ldots, h_n^{enc}$ and h_t^{dec} are hidden states from encoder and decoder RNNs



- Encoder hidden states: $h_1^{enc}, \dots, h_n^{enc}$ (n: # of words in source sentence)
- Decoder hidden state at time t: h_t^{dec}
- Attention scores:

$$e^t = [g(h_1^{enc}, h_t^{dec}), \dots, g(h_n^{enc}, h_t^{dec})] \in \mathbb{R}^n$$

Attention distribution:

$$\alpha^t = \operatorname{softmax}(e^t) \in \mathbb{R}^n$$

Weighted sum of encoder hidden states:

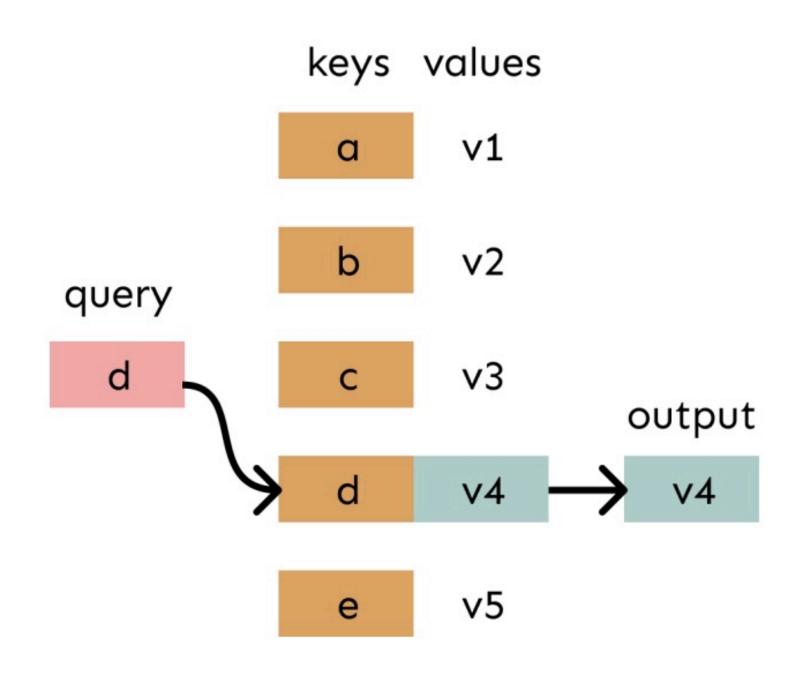
$$o_t = \sum_{i=1}^n \alpha_i^t h_i^{enc} \in \mathbb{R}^h$$

Combine o_t and h_t^{dec} to predict next word

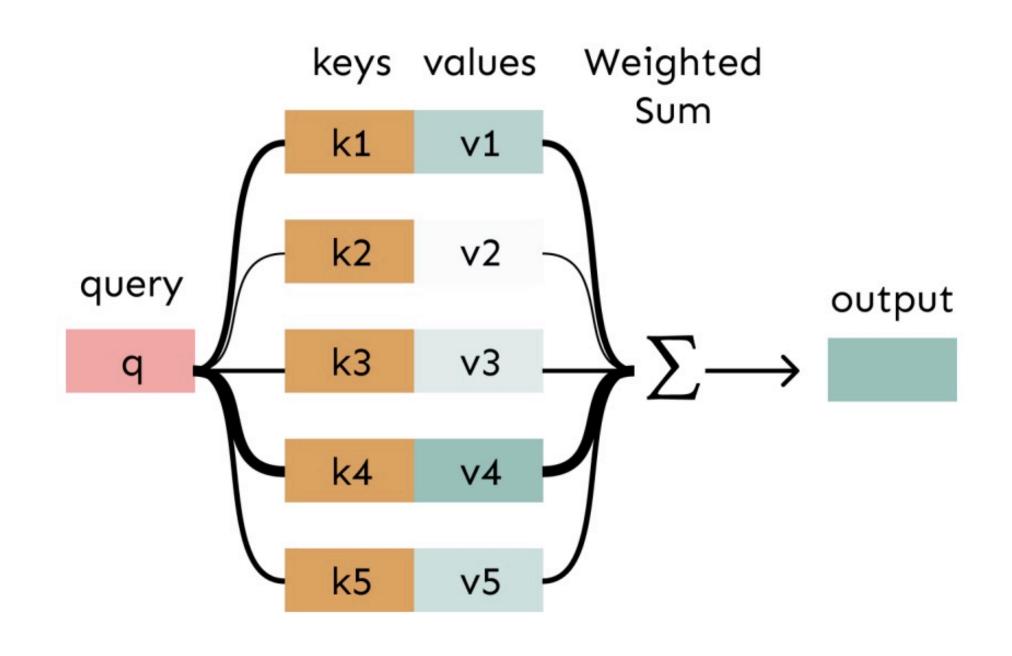
Attention as a soft, averaging lookup table

We can think of attention as performing fuzzy lookup a in key-value store

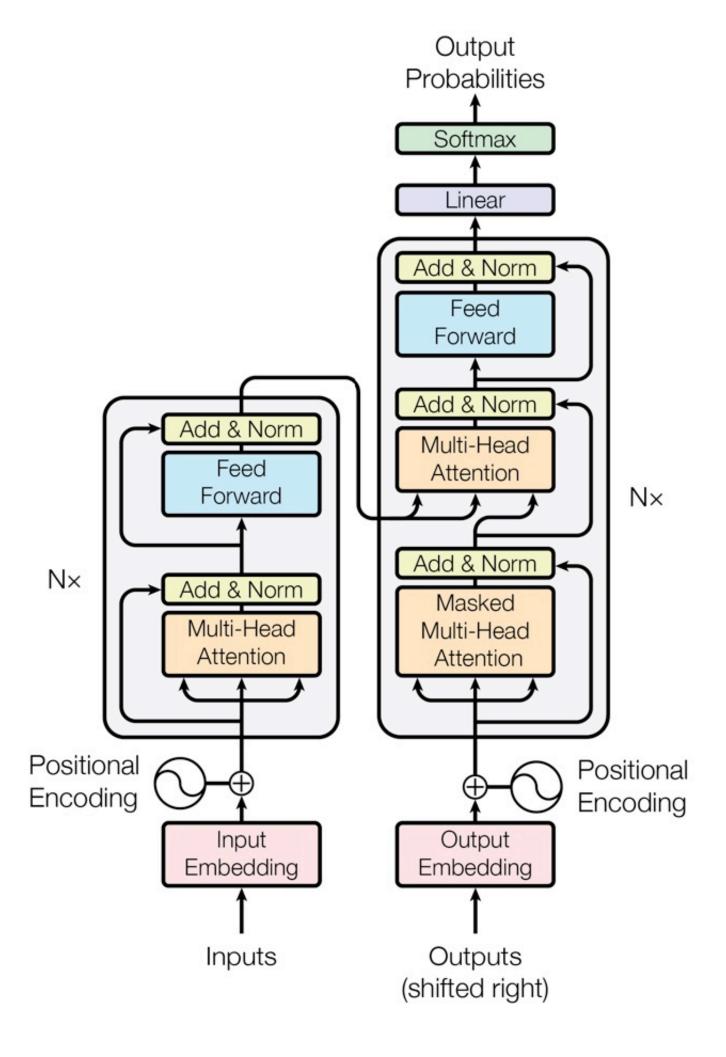
Lookup table: a table of keys that map to values. The query matches one of the keys, returning its value.



Attention: The query matches to all keys softly to a weight between 0 and 1. The keys' values are multiplied by the weights and summed.

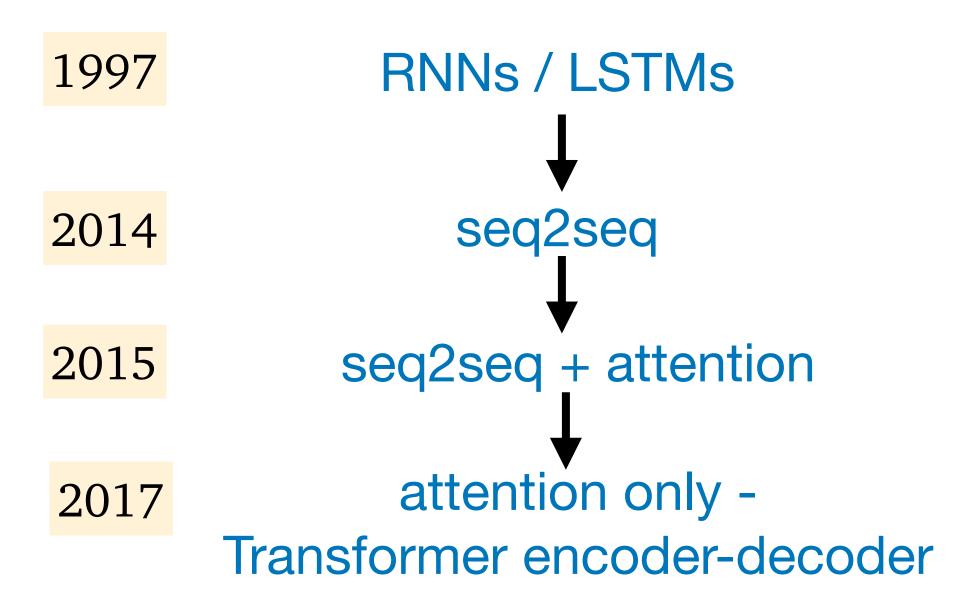


Transformer encoder-decoder



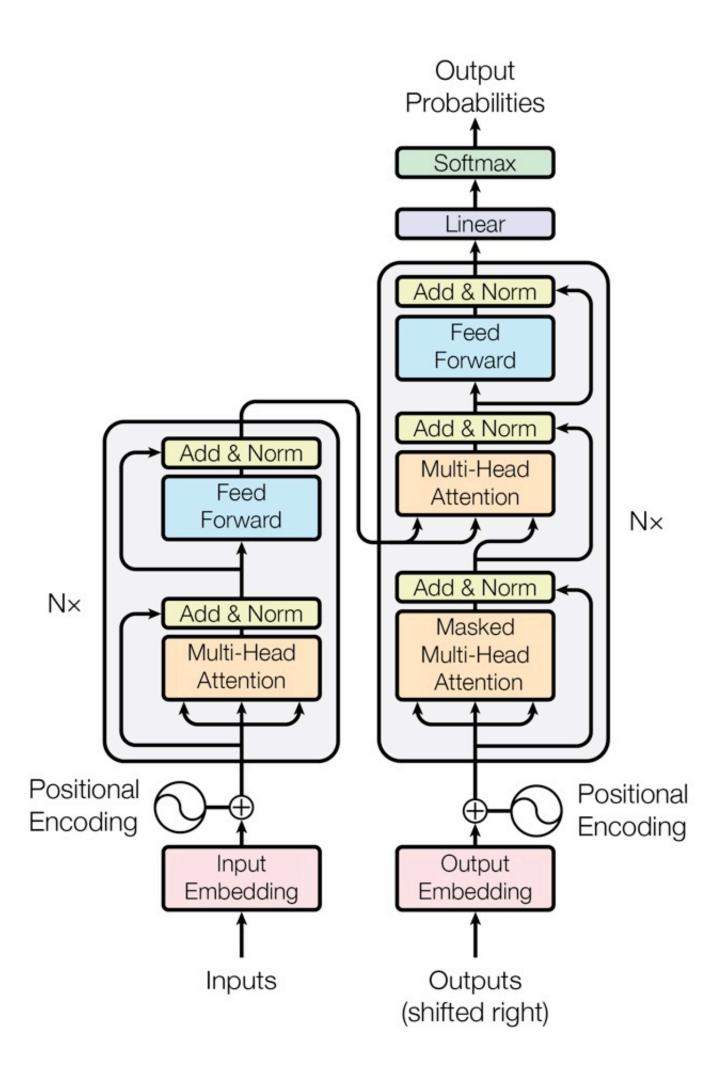
(Vaswani et al., 2017)

- Transformer encoder + Transformer decoder: a replacement for seq2seq + attention based on RNNs
- First designed and experimented on NMT



Transformers (both encoders and decoders) have become the default neural architectures in modeling languages!

Transformer encoder-decoder



- Transformer encoder = a stack of encoder layers
- Transformer decoder = a stack of decoder layers

Transformer encoder: BERT, RoBERTa, ELECTRA

Transformer decoder: GPT-n, ChatGPT, Gemini,

Claude, LLaMA, Mistral, ...

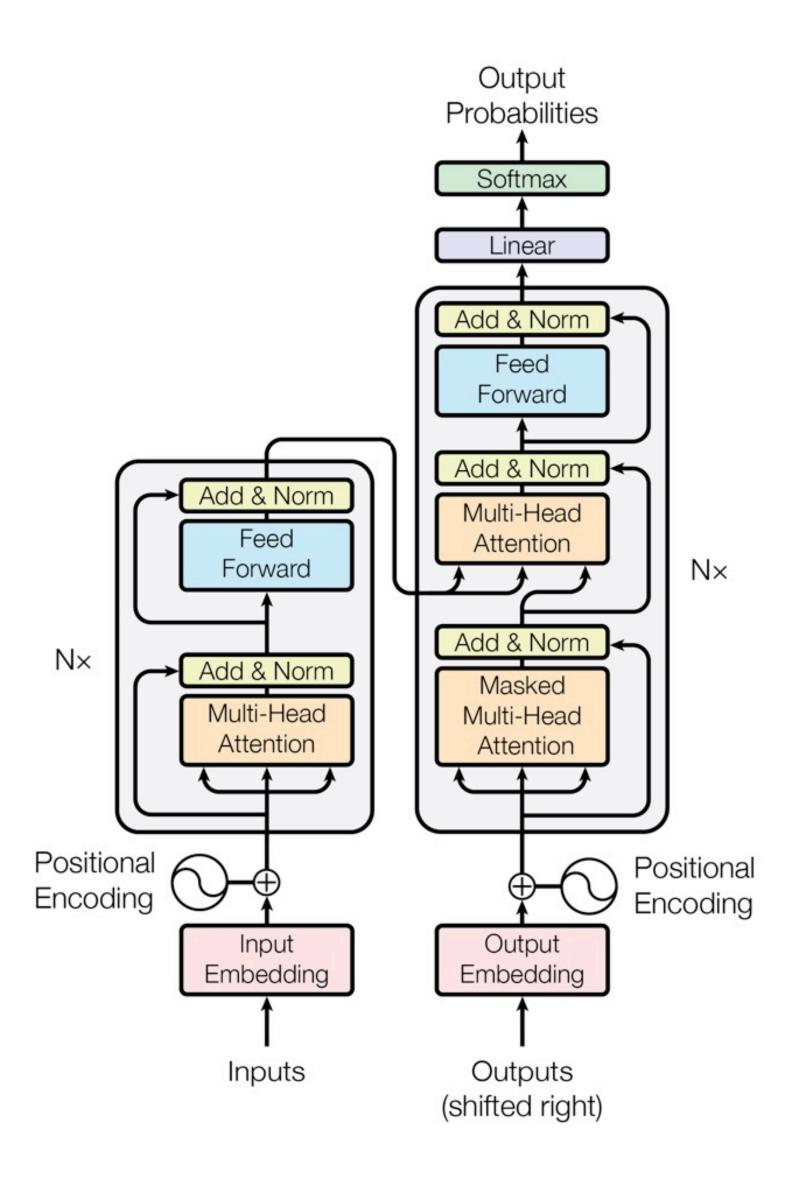
Transformer encoder-decoder: T5, BART

- Key innovation: self-attention, multi-head
- Transformers don't have any recurrence structures!

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t) \in \mathbb{R}^h$$

(Vaswani et al., 2017)

Transformers: roadmap



- Self-attention and multi-head attention
- Feedforward layers
- Positional encoding
- Residual connections + layer normalization
- Transformer encoder vs Transformer decoder

The Annotated Transformer

Attention is All You Need

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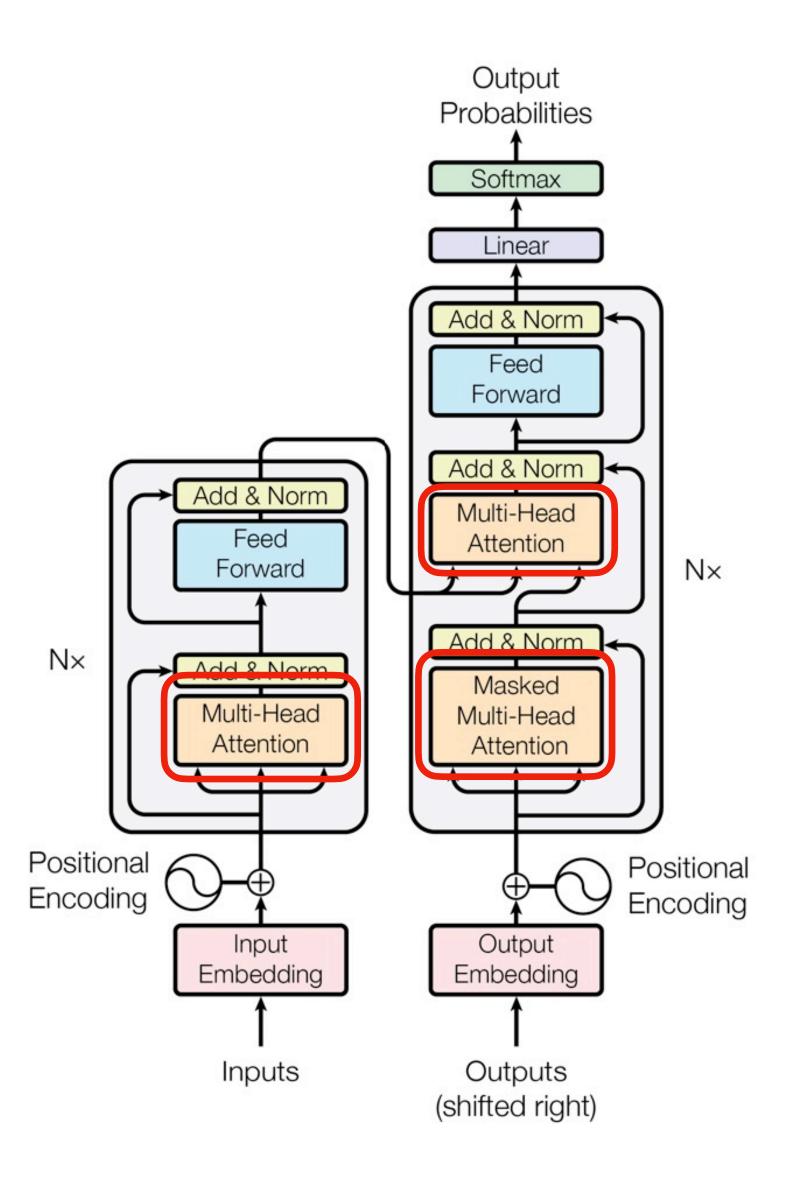
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https://nlp.seas.harvard.edu/annotated-transformer/

Transformers: roadmap



- Self-attention and multi-head attention
- 4

- Feedforward layers
- Positional encoding
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- Advanced techniques: SwiGLU, rotary embeddings, pre-normalization, grouped query attention
- Architecture exploration beyond Transformers

General form of attention

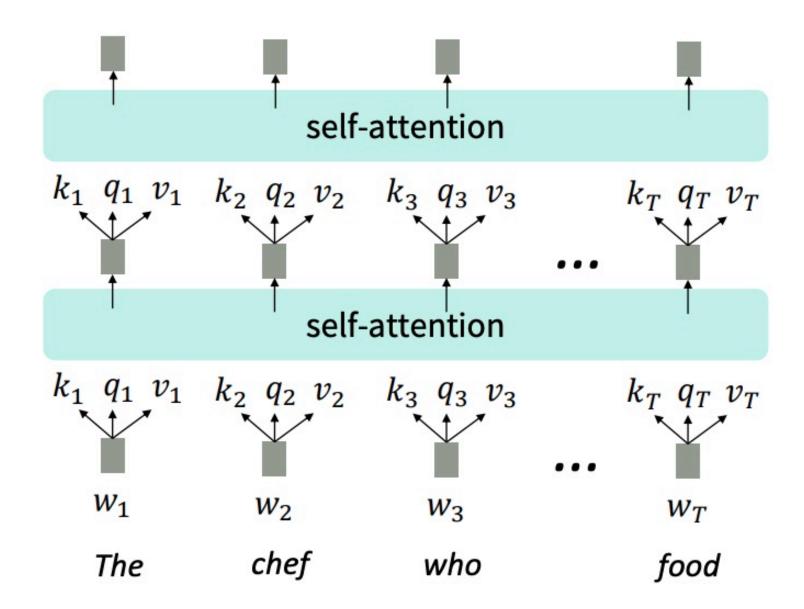
- A more general form: use a set of keys and values $(\mathbf{k}_1, \mathbf{v}_1), \ldots, (\mathbf{k}_n, \mathbf{v}_n), \mathbf{k}_i \in \mathbb{R}^{d_k}, \mathbf{v}_i \in \mathbb{R}^{d_v}$, keys are used to compute the attention scores and values are used to compute the output vector
- Attention always involves the following steps:
 - Computing the attention scores $\mathbf{e} = g(\mathbf{q}, \mathbf{k}_i) \in \mathbb{R}^n$
 - Taking softmax to get attention distribution α :

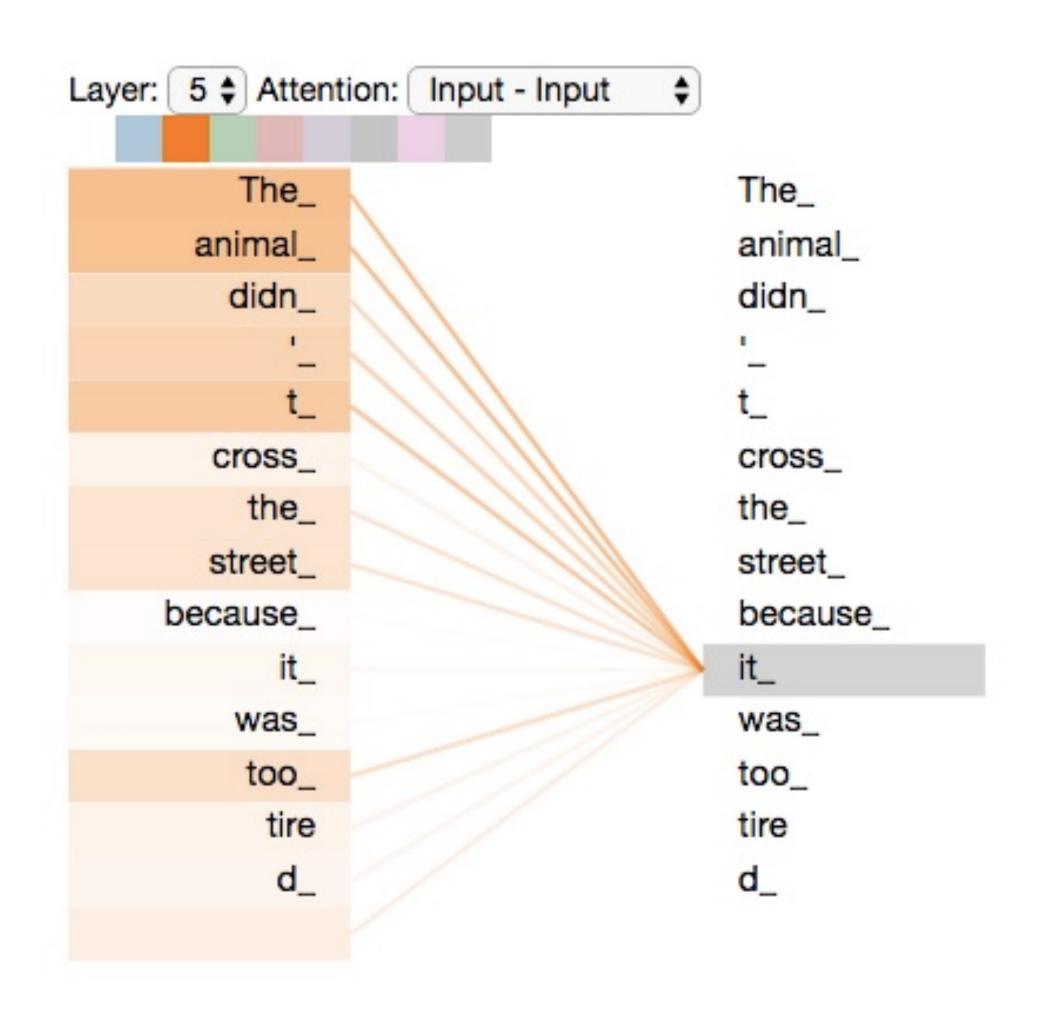
$$\alpha = \operatorname{softmax}(\mathbf{e}) \in \mathbb{R}^n$$

Using attention distribution to take weighted sum of values:

$$\mathbf{o} = \sum_{i=1}^{m} \alpha_i \mathbf{v}_i \in \mathbb{R}^{d_v}$$

- In NMT, query = decoder's hidden state,
 keys = values = encoder's hidden states
- Self-attention = attention from the sequence to **itself**
- Self-attention: let's use each word in a sequence as the query, and all other words in the sequence as keys and values.

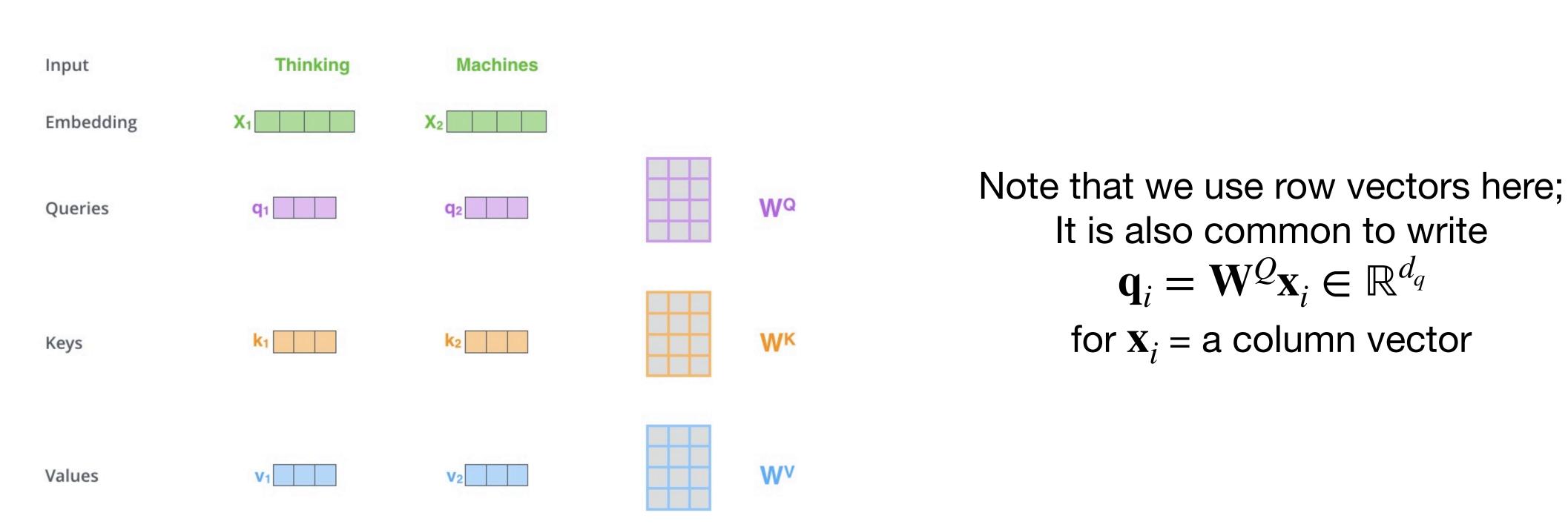




Step #1: Transform each input vector into three vectors: query, key, and value vectors

$$\mathbf{q}_{i} = \mathbf{x}_{i} \mathbf{W}^{Q} \in \mathbb{R}^{d_{q}} \qquad \mathbf{k}_{i} = \mathbf{x}_{i} \mathbf{W}^{K} \in \mathbb{R}^{d_{k}} \qquad \mathbf{v}_{i} = \mathbf{x}_{i} \mathbf{W}^{V} \in \mathbb{R}^{d_{v}}$$

$$\mathbf{W}^{Q} \in \mathbb{R}^{d_{in} \times d_{q}} \qquad \mathbf{W}^{K} \in \mathbb{R}^{d_{in} \times d_{k}} \qquad \mathbf{W}^{V} \in \mathbb{R}^{d_{in} \times d_{v}}$$



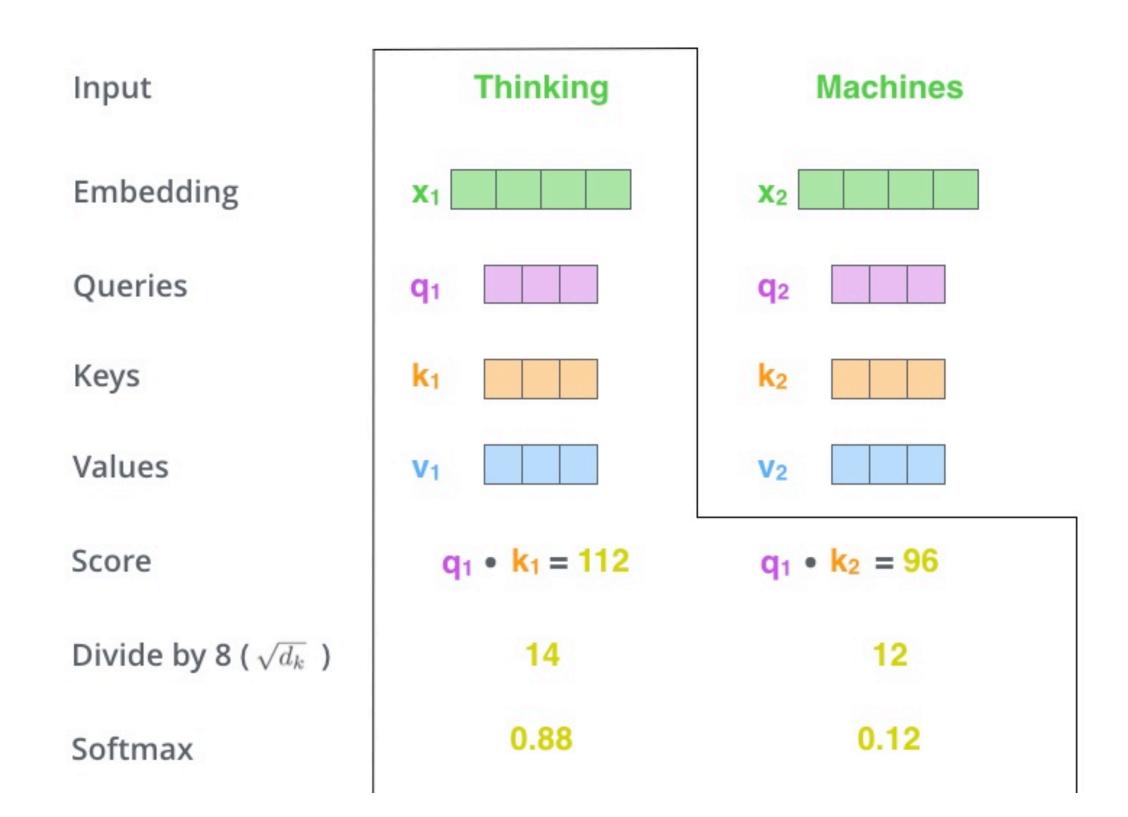
Step #2: Compute pairwise similarities between keys and queries; normalize with softmax For each \mathbf{q}_i , compute attention scores and attention distribution:

$$\alpha_{i,j} = \operatorname{softmax}(\frac{\mathbf{q}_i \cdot \mathbf{k}_j}{\sqrt{d_k}})$$

aka. "scaled dot product" It must be $d_q = d_k$ in this case

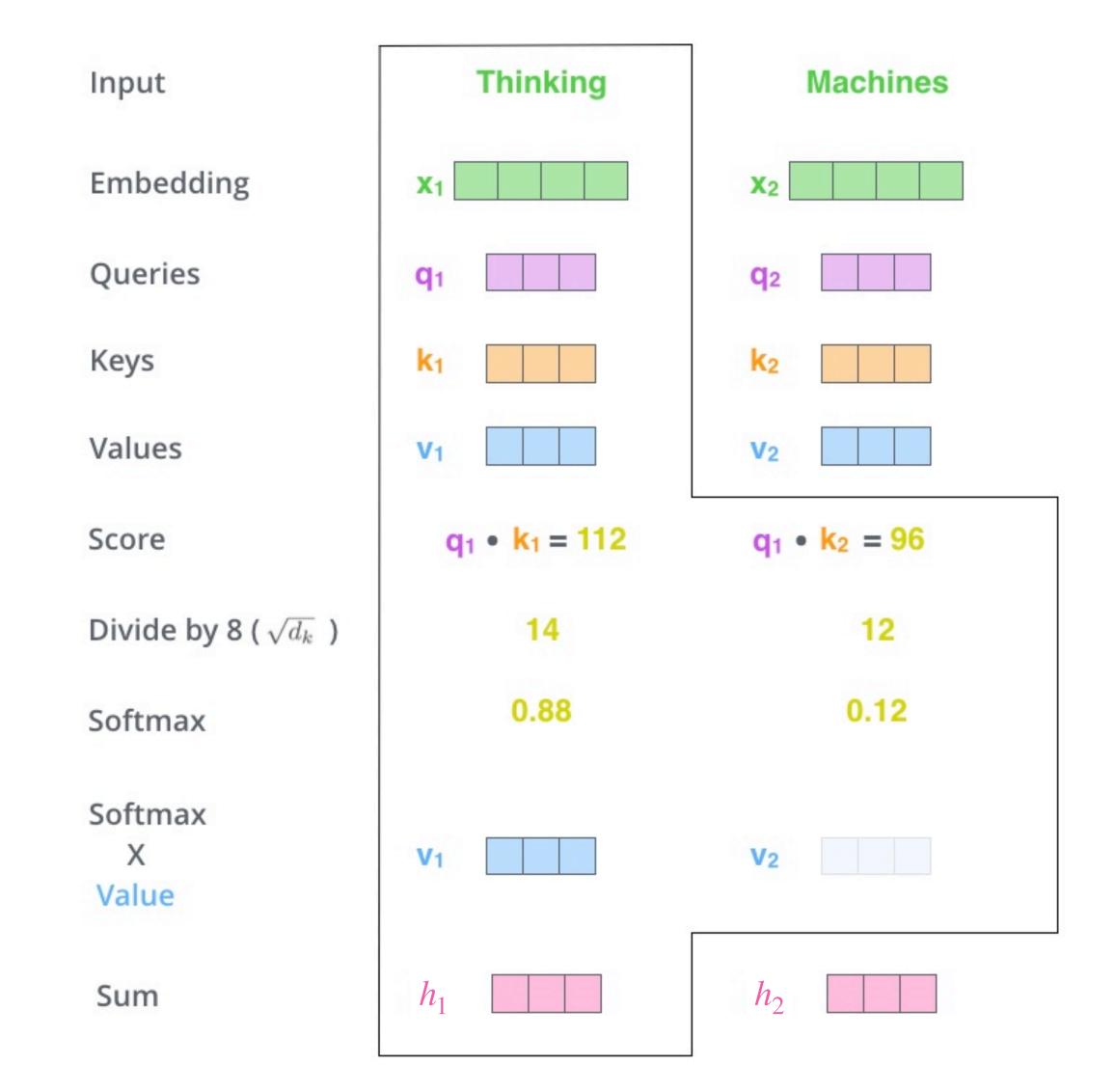
Q. Why scaled dot product?

To avoid the dot product to become too large for larger d_k ; scaling the dot product by $\frac{1}{\sqrt{d_k}}$ is easier for optimization



Step #3: Compute output for each input as weighted sum of values

$$\mathbf{h}_i = \sum_{j=1}^n \alpha_{i,j} \mathbf{v}_j \in \mathbb{R}^{d_v}$$



https://jalammar.github.io/illustrated-transformer/



What would be the output vector for the word "Thinking" approximately?

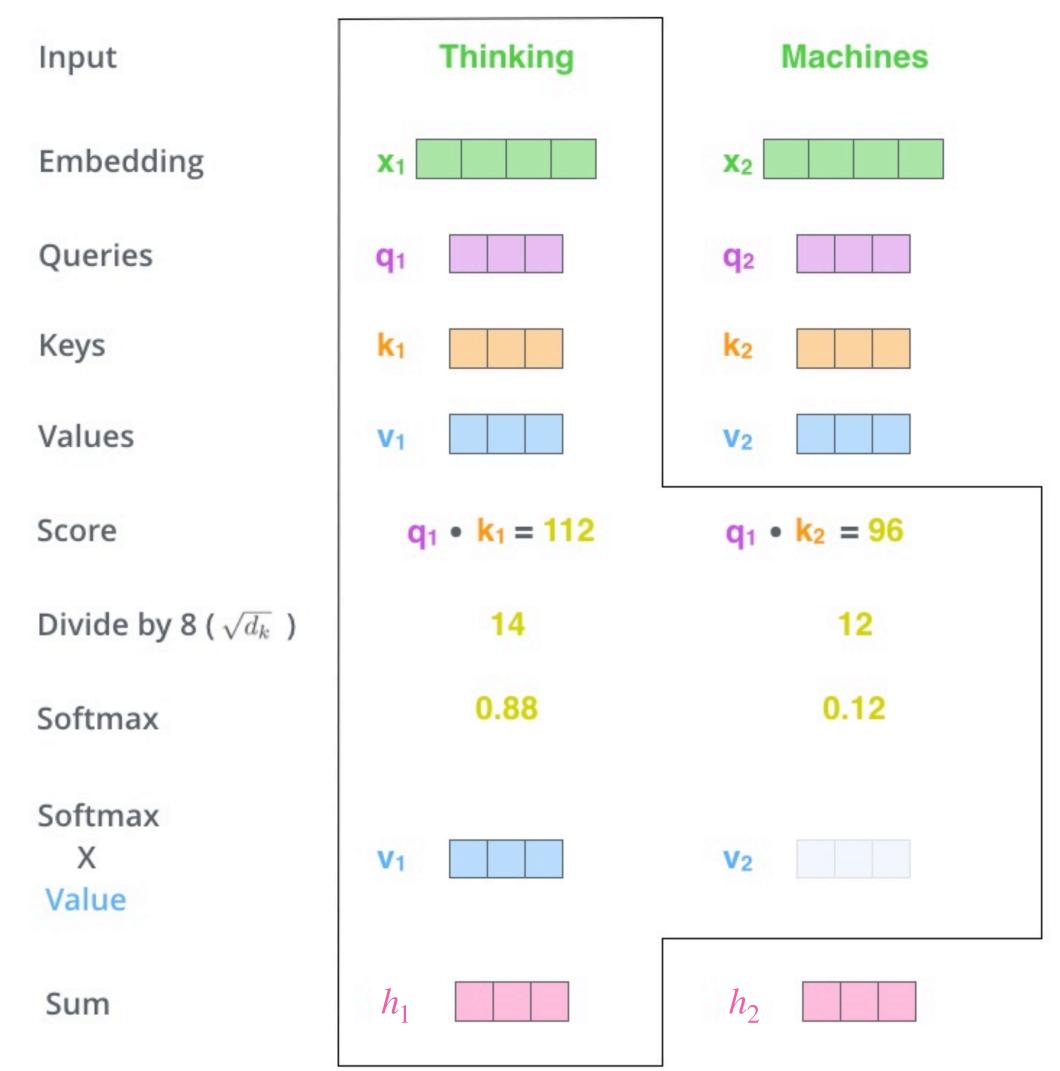
(A)
$$0.5\mathbf{v}_1 + 0.5\mathbf{v}_2$$

(B)
$$0.54\mathbf{v}_1 + 0.46\mathbf{v}_2$$

(C)
$$0.88\mathbf{v}_1 + 0.12\mathbf{v}_2$$

(D)
$$0.12\mathbf{v}_1 + 0.88\mathbf{v}_2$$

(C) is correct.

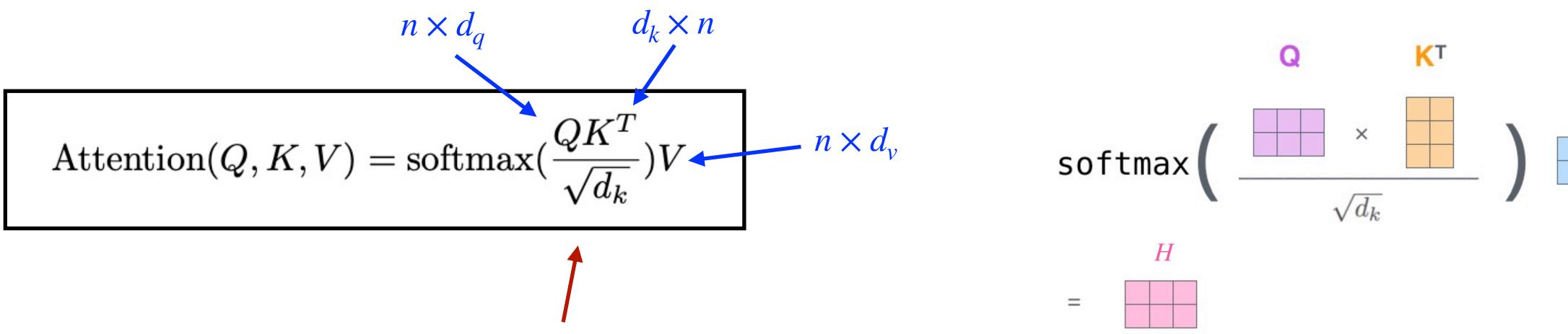


Self-attention: matrix notations

$$X \in \mathbb{R}^{n \times d_{in}}$$
 (n = input length)

$$Q = XW^Q, K = XW^K, V = XW^V$$

where
$$W^Q \in \mathbb{R}^{d_{in} \times d_q}$$
, $W^K \in \mathbb{R}^{d_{in} \times d_k}$, $W^V \in \mathbb{R}^{d_{in} \times d_v}$

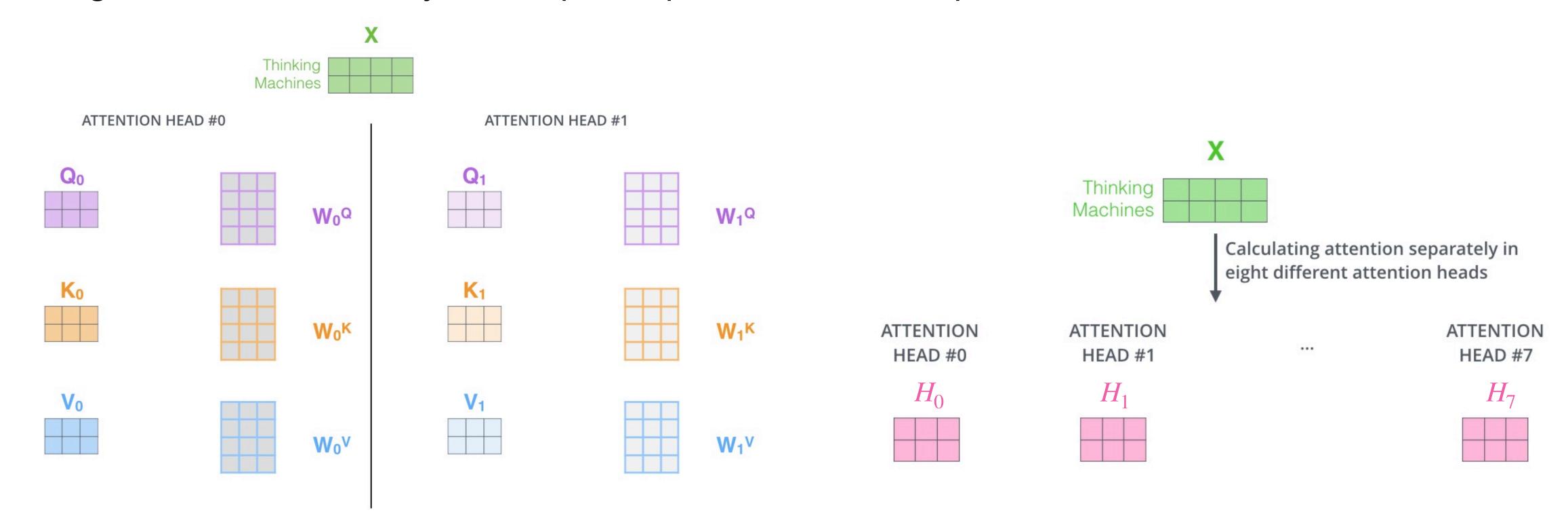


Q: What is this softmax operation?

Multi-head attention

"The Beast with Many Heads"

- It is better to use multiple attention functions instead of one!
 - Each attention function ("head") can focus on different positions.
- It gives the attention layer multiple "representation subspaces"

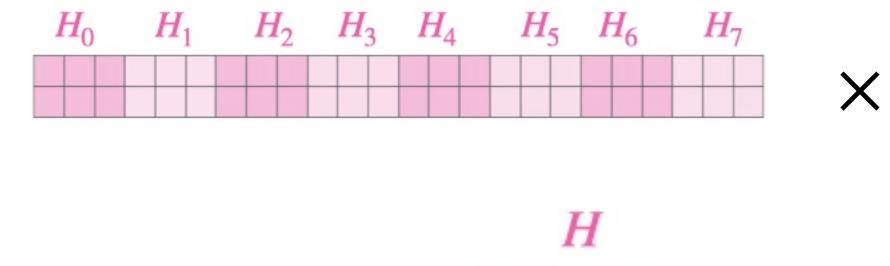


Multi-head attention

"The Beast with Many Heads"

Finally, we just concatenate all the heads and apply an output projection matrix.

$$\begin{aligned} \text{MultiHead}(Q, K, V) &= \text{Concat}(\text{head}_1, ..., \text{head}_h) W^O \\ \text{head}_i &= \text{Attention}(XW_i^Q, XW_i^K, XW_i^V) \end{aligned}$$



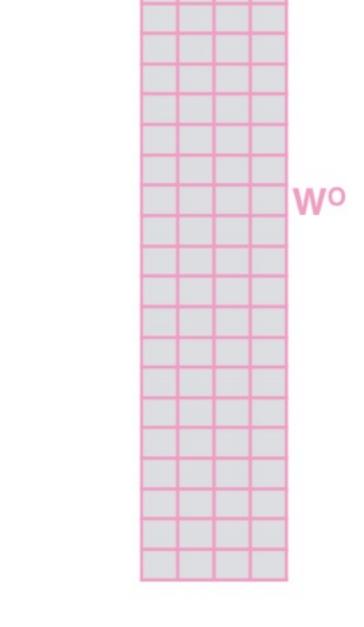
• In practice, we use a *reduced* dimension for each head.

$$W_i^Q \in \mathbb{R}^{d_{in} \times d_q}, W_i^K \in \mathbb{R}^{d_{in} \times d_k}, W_i^V \in \mathbb{R}^{d_{in} \times d_v}$$

$$d_q = d_k = d_v = d/m \quad d = \text{hidden size}, m = \# \text{ of heads}$$

$$W^O \in \mathbb{R}^{d \times d_{out}}$$

 The total computational cost is similar to that of single-head attention with full dimensionality.



Multi-head attention

"The Beast with Many Heads"

$$\begin{aligned} \text{MultiHead}(Q, K, V) &= \text{Concat}(\text{head}_1, ..., \text{head}_{\text{h}}) W^O \\ \text{head}_i &= \text{Attention}(XW_i^Q, XW_i^K, XW_i^V) \end{aligned}$$

• We can think of multi-head attention (MHA) layer as an abstraction layer that maps a sequence of input vectors $\mathbf{x}_1, ..., \mathbf{x}_n \in \mathbb{R}^{d_{in}}$ to a sequence of n vectors: $\mathbf{h}_1, ..., \mathbf{h}_n \in \mathbb{R}^{d_{out}}$

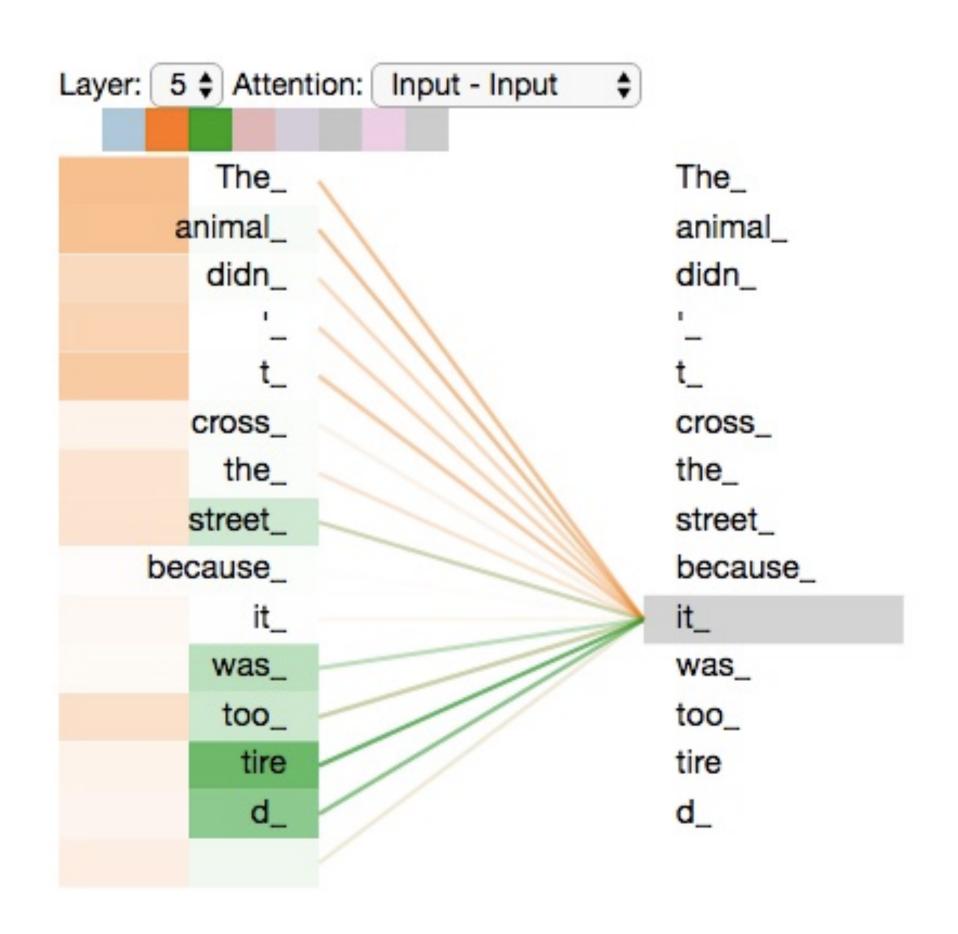
If we stack multiple layers, usually $d_{in}=d_{out}=d$

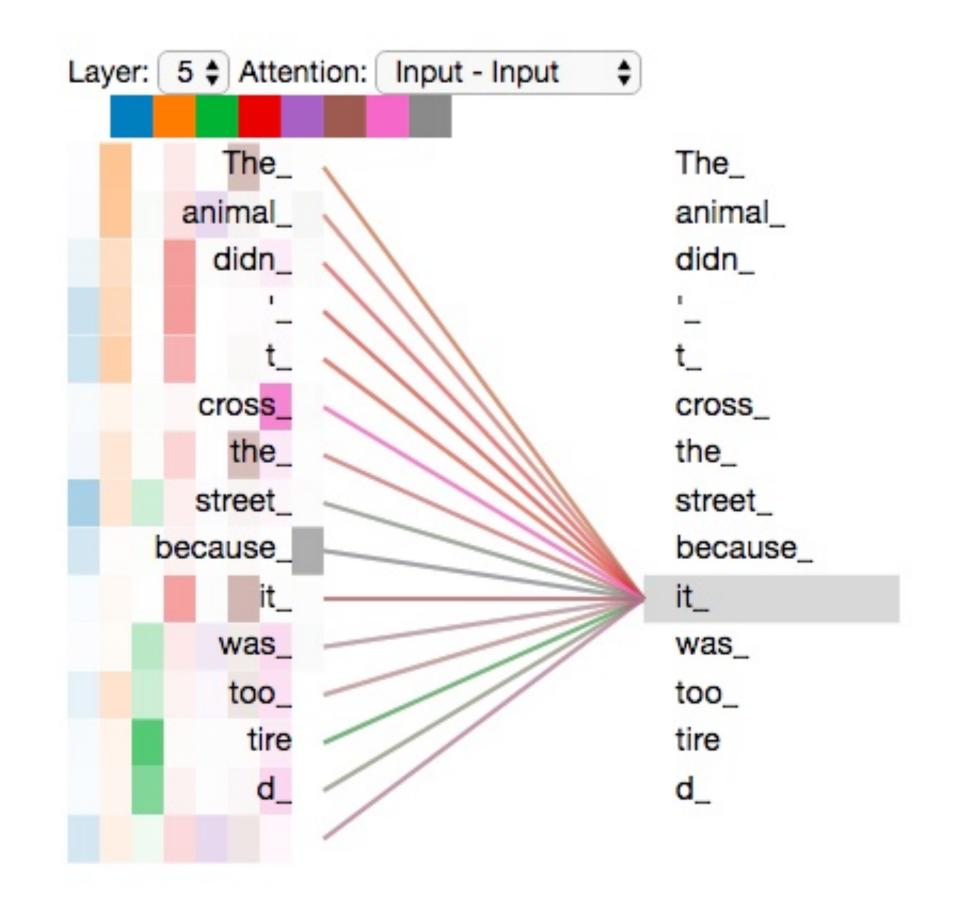
The same abstraction as RNNs - used as a drop-in replacement for an RNN layer

$$\mathbf{h}_t = f(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_t + \mathbf{b}) \in \mathbb{R}^{d_{out}}$$

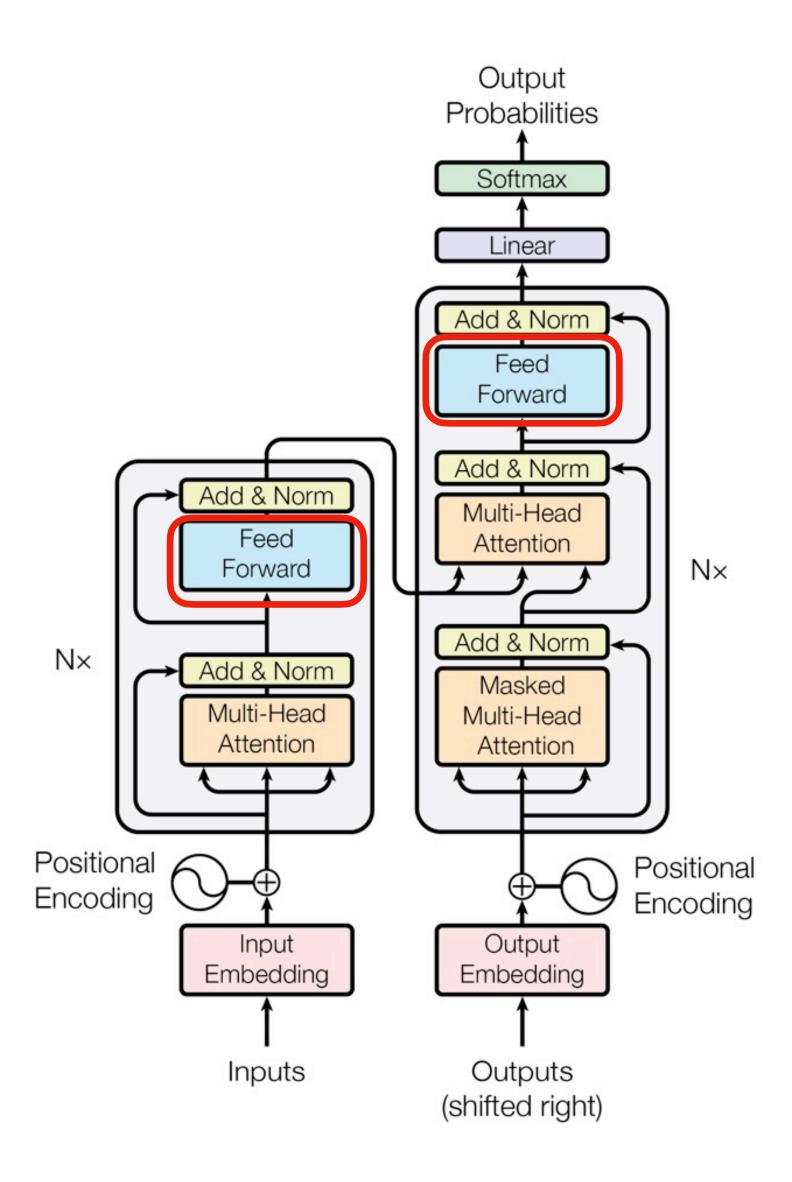
Much easier to parallelize, more expensive to scale up to longer sequences!

What does multi-head attention learn?





Transformers: roadmap

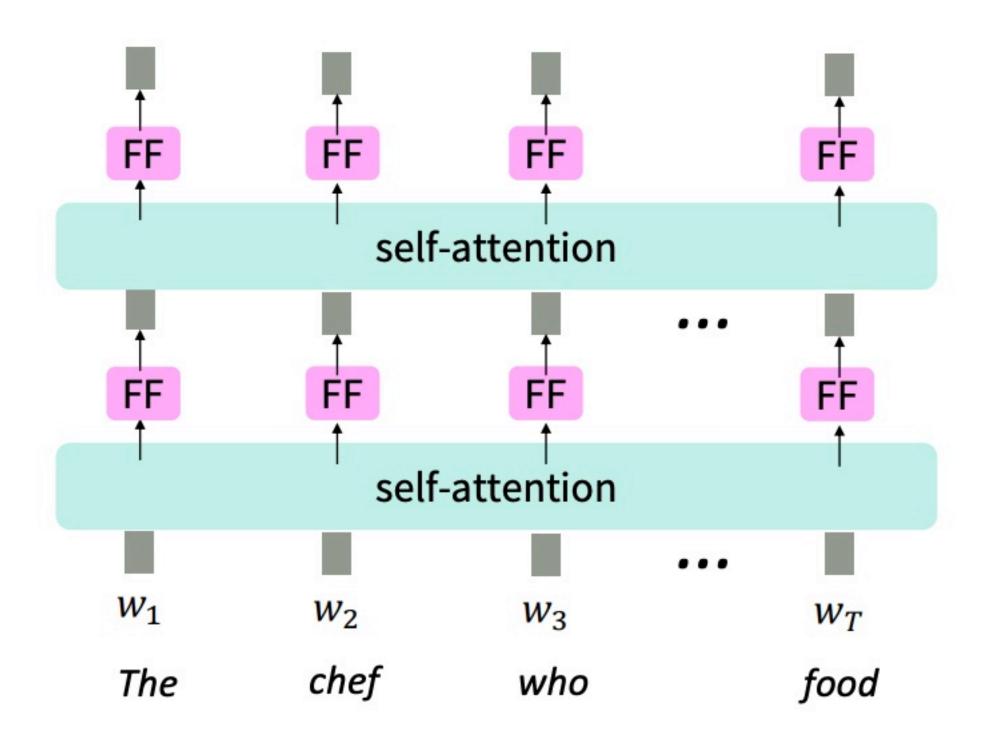


- Self-attention and multi-head self-attention
- Feedforward layers
- Positional encoding
- Residual connections + layer normalization
- Transformer encoder vs Transformer decoder
- Advanced techniques: SwiGLU, rotary embeddings, pre-normalization, grouped query attention
- Architecture exploration beyond Transformers

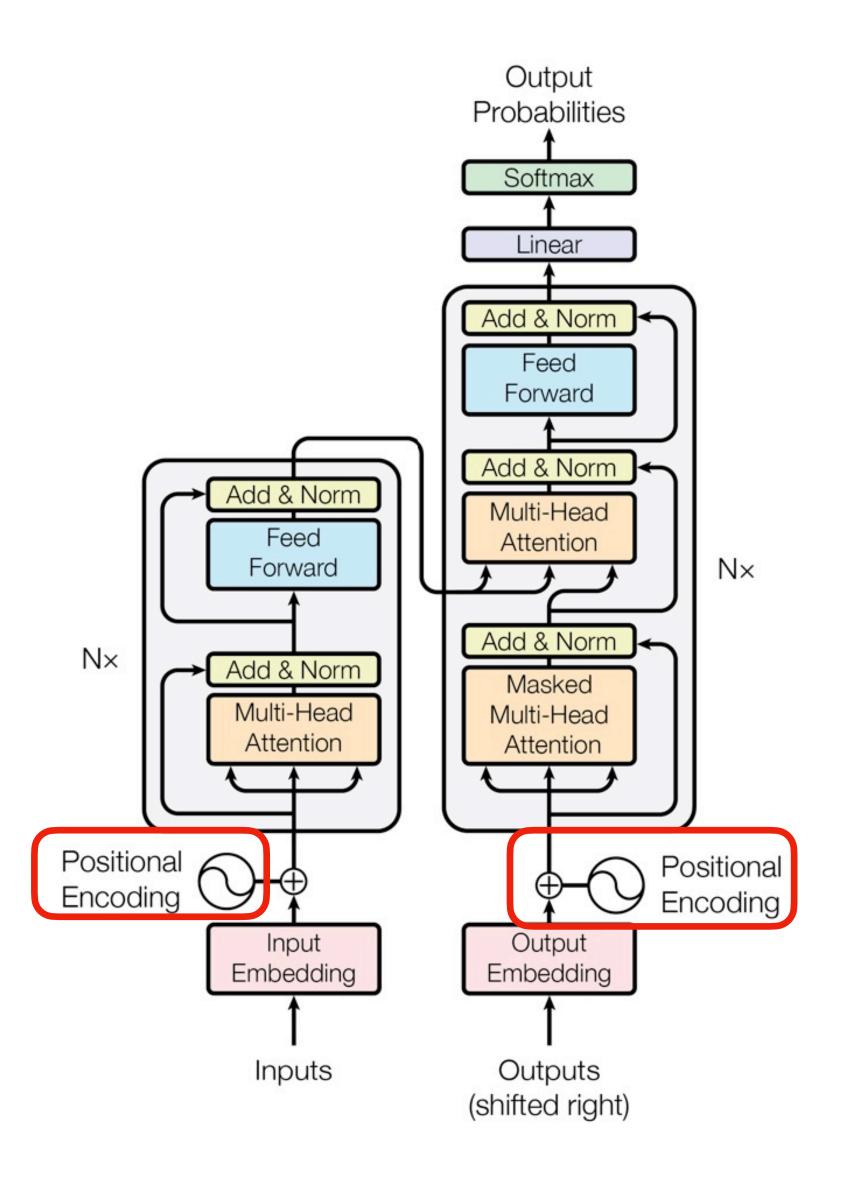
Adding nonlinearities: Feed-forward layers

- There are no element-wise nonlinearities in self-attention; stacking more self-attention layers just re-averages value vectors
- Simple fix: add a feed-forward network to post-process each output vector

$$ext{FFN}(\mathbf{x}_i) = ext{ReLU}(\mathbf{x}_i \mathbf{W}_1 + \mathbf{b}_1) \mathbf{W}_2 + \mathbf{b}_2$$
 $\mathbf{W}_1 \in \mathbb{R}^{d imes d_{ff}}, \mathbf{b}_1 \in \mathbb{R}^{d_{ff}}$ $\mathbf{W}_2 \in \mathbb{R}^{d_{ff} imes d}, \mathbf{b}_2 \in \mathbb{R}^d$ Usually, $d_{ff} = 4d$



Transformers: roadmap



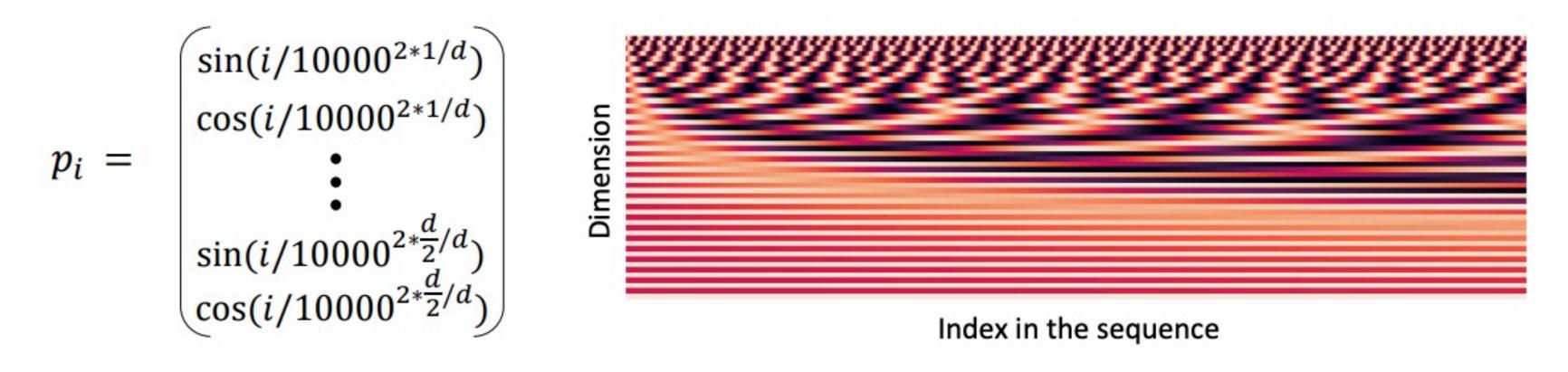
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Modeling order information: positional encoding

- Unlike RNNs, self-attention doesn't build in order information, we need to encode the order
 of the sentence in our keys, queries, and values
- Solution: Add positional embeddings to the input embeddings: $\mathbf{p}_i \in \mathbb{R}^d$ for $i=1,2,\ldots,n$

$$\mathbf{x}_i \leftarrow \mathbf{x}_i + \mathbf{p}_i$$

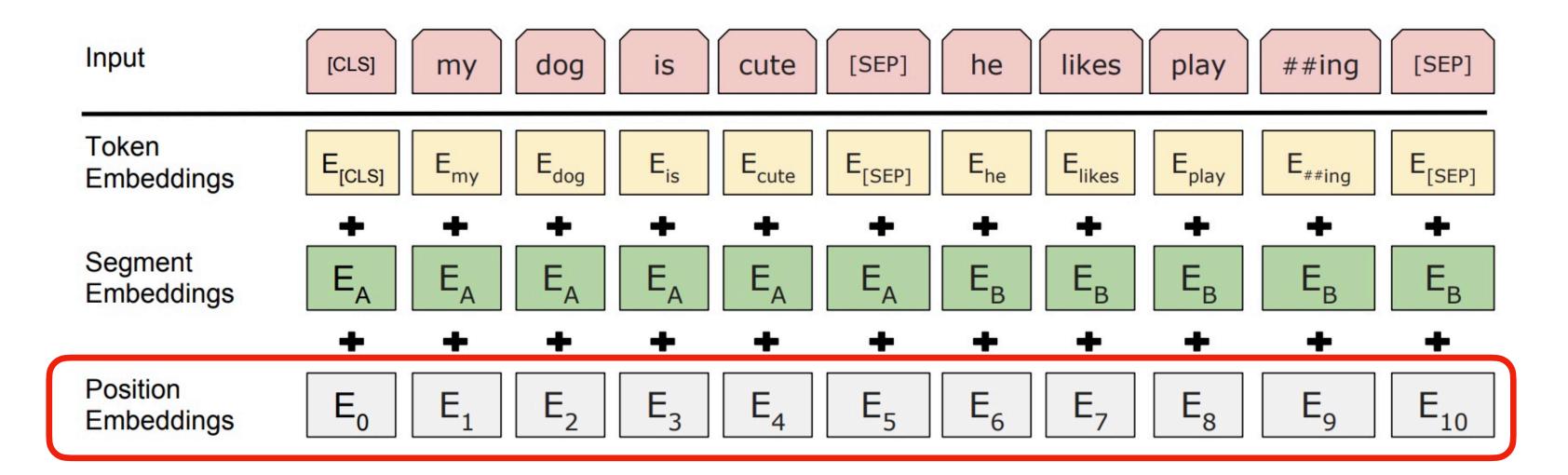
• Sinusoidal positional embeddings: sine and cosine functions of different frequencies:



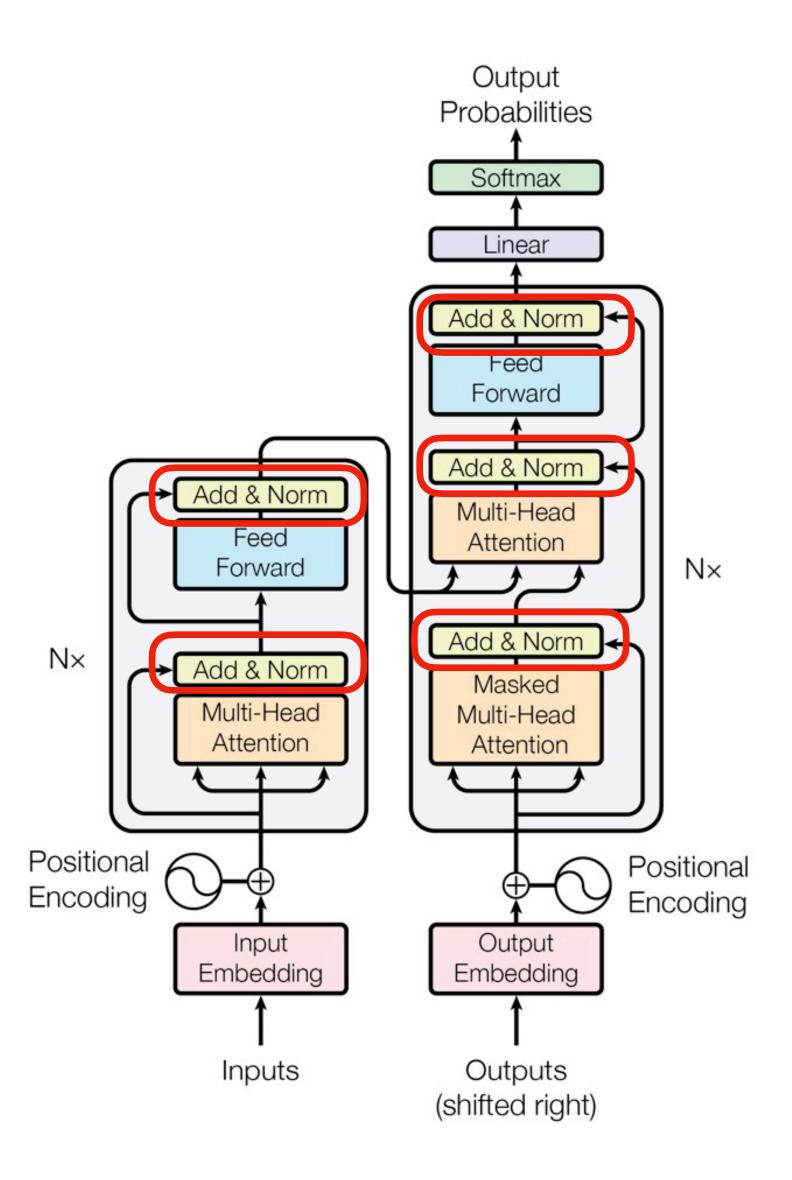
- Pros: Periodicity + can extrapolate to longer sequences
- Cons: Not learnable

Modeling order information: positional encoding

- Absolute positional embeddings: let all \mathbf{p}_i be learnable parameters
 - $P \in \mathbb{R}^{d \times L}$ for $L = \max$ sequence length
 - Pros: each position gets to be learned to fit the data
 - Cons: can't extrapolate to indices outside of max sequence length L
 - Examples: BERT, GPT-1, GPT-2, GPT-3, OPT



Transformers: roadmap



- Self-attention and multi-head attention
- Feedforward layers
- Positional encoding
- Residual connections + layer normalization



- Transformer encoder vs Transformer decoder
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How to make Transformers work for deep NNs?

Add & Norm: LayerNorm(x + Sublayer(x))

Residual connections (He et al., CVPR 2016)

Instead of $X^{(i)} = \text{Layer}(X^{(i-1)})$ (*i* represents the layer)

$$X^{(i-1)}$$
 — Layer $X^{(i)}$

We let $X^{(i)} = X^{(i-1)} + \text{Layer}(X^{(i-1)})$, so we only need to learn "the residual" from the previous layer

$$X^{(i-1)}$$
 Layer $X^{(i)}$

This prevents the network from "forgetting" or distorting important information as it is processed by many layers.

How to make Transformers work for deep NNs?

Add & Norm: LayerNorm(x + Sublayer(x))

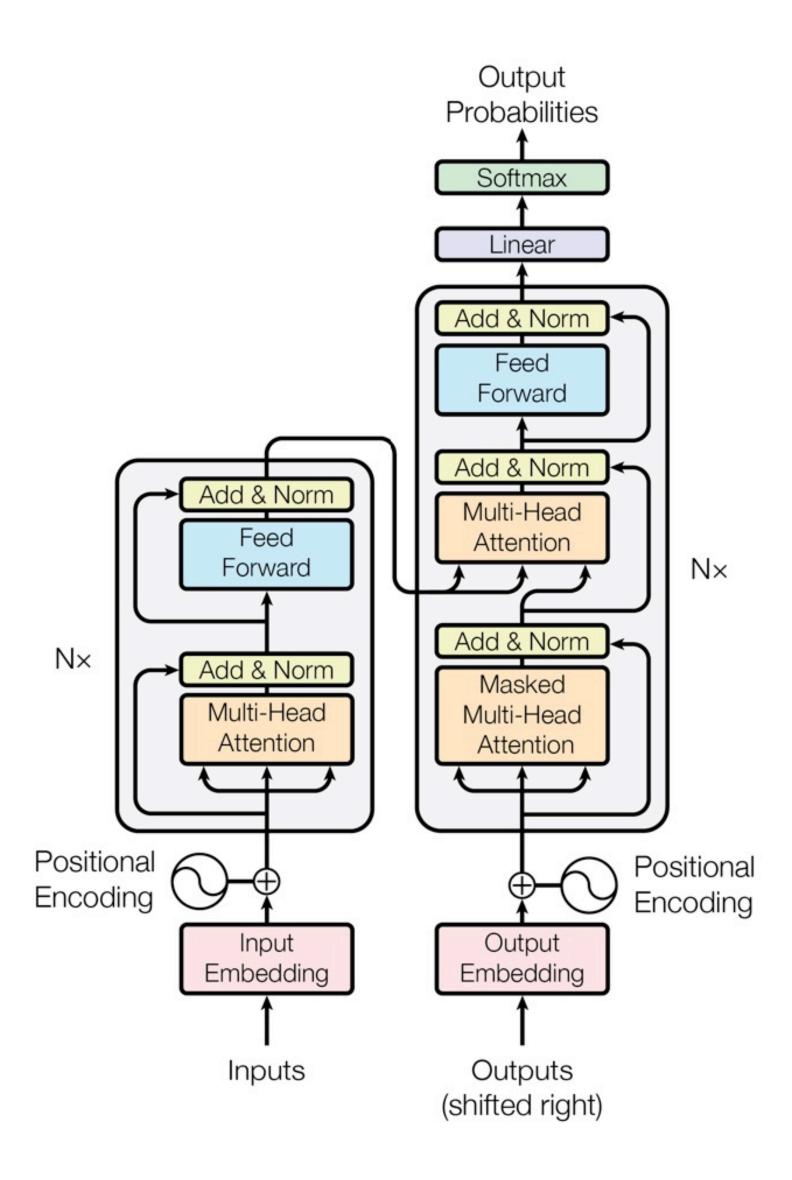
Layer normalization (Ba et al., 2016)

Problem: Difficult to train the parameters of a given layer because its input from the layer beneath keeps shifting.

Solution: Reduce variation by **normalizing** to zero mean and standard deviation of one **within each layer**.

$$y = rac{x - \mathrm{E}[x]}{\sqrt{\mathrm{Var}[x] + \epsilon}} * \gamma + eta \qquad \gamma, eta \in \mathbb{R}^d$$
 are learnable parameters

Transformers: roadmap

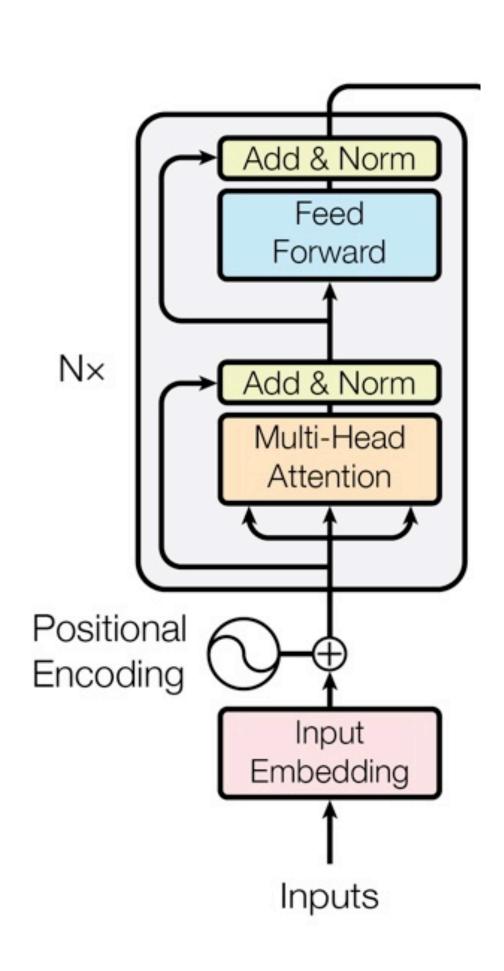


- Self-attention and multi-head attention
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- Positional encoding
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- Transformer encoder vs Transformer decoder



- Advanced techniques: SwiGLU, rotary embeddings, pre-normalization, grouped query attention
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Let's put things together - Transformer encoder



From the bottom to the top:

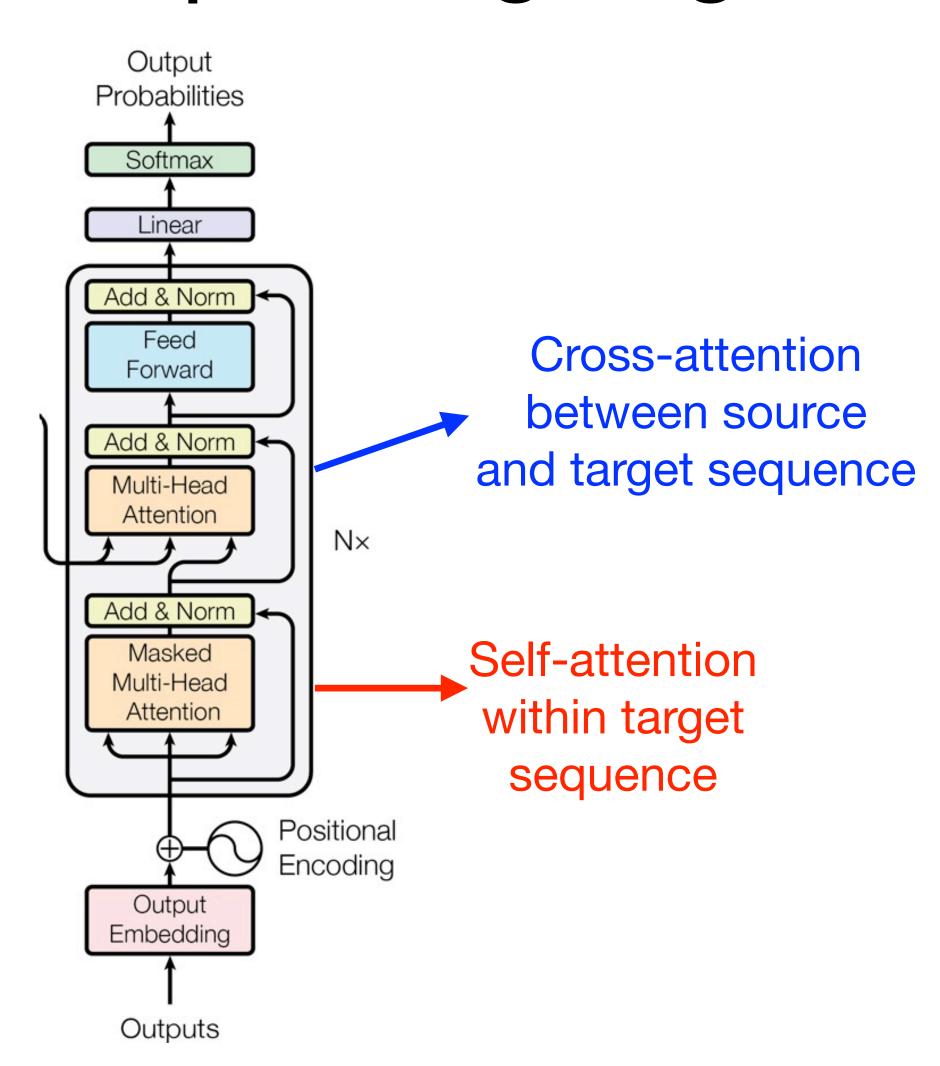
- Input embedding
- Positional encoding
- A stack of Transformer encoder layers

Transformer encoder is a stack of N layers, which consists of two sub-layers:

- Multi-head attention layer
- Feed-forward layer

$$\mathbf{x}_1, ..., \mathbf{x}_n \in \mathbb{R}^{d_{in}} \longrightarrow \mathbf{h}_1, ..., \mathbf{h}_n \in \mathbb{R}^{d_{out}}$$

Let's put things together - Transformer decoder



From the bottom to the top:

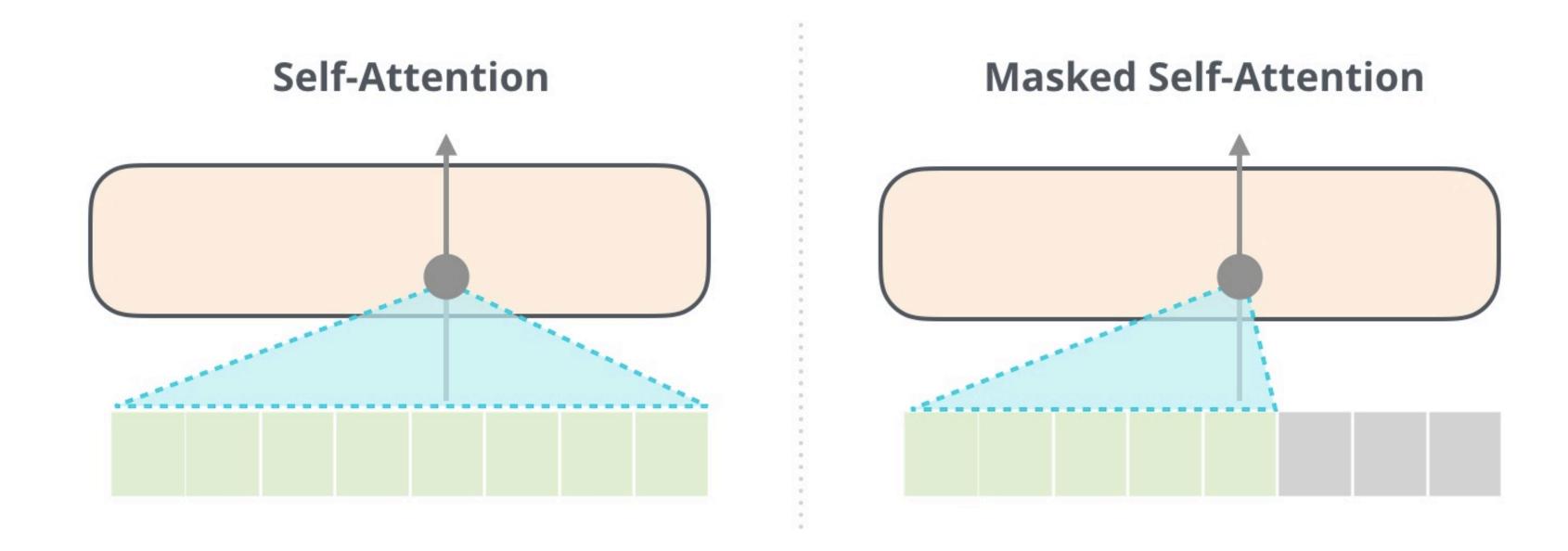
- Output embedding
- Positional encoding
- A stack of Transformer decoder layers
- Linear + softmax

Transformer decoder is a stack of N layers, which consists of **three** sub-layers:

- Masked multi-head attention
- Multi-head cross-attention
- Feed-forward layer
- (w/ Add & Norm between sub-layers)

Masked multi-head self-attention

Key: You can't see the future text for the decoder!



• Solution: for every q_i , only attend to $\{(k_j, v_j)\}, j \leq i$ How to implement this? Masking!

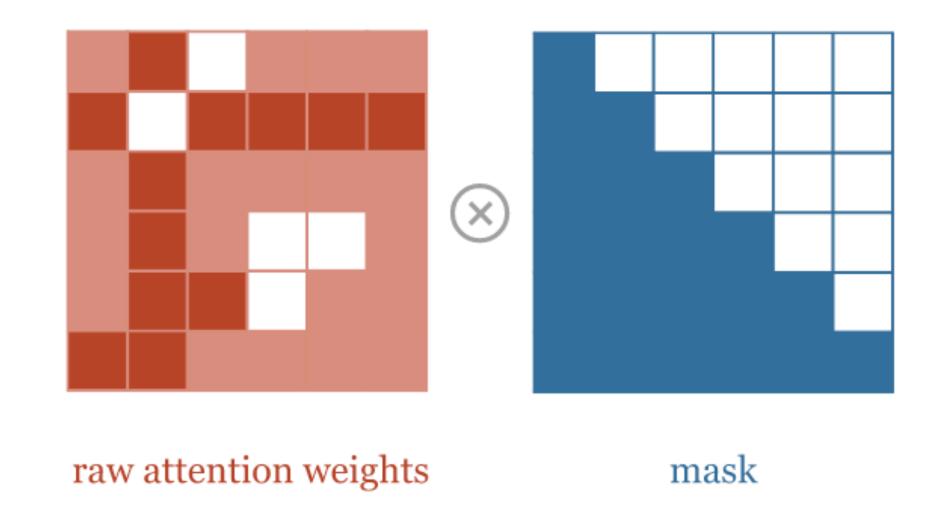
Masked multi-head self-attention

$$\mathbf{q}_{i} = \mathbf{x}_{i} \mathbf{W}^{Q}, \mathbf{k}_{i} = \mathbf{x}_{i} \mathbf{W}^{K}, \mathbf{v}_{i} = \mathbf{x}_{i} \mathbf{W}^{V}$$

$$e_{i,j} = \frac{\mathbf{q}_{i} \cdot \mathbf{k}_{j}}{\sqrt{d_{k}}}, \forall j = 1, \dots, n$$

$$\alpha_{i} = \operatorname{softmax}(\mathbf{e}_{i})$$

Efficient implementation: compute attention as we normally do, mask out attention to future words by setting attention scores to $-\infty$



```
dot = torch.bmm(queries, keys.transpose(1, 2))
indices = torch.triu_indices(t, t, offset=1)
dot[:, indices[0], indices[1]] = float('-inf')

dot = F.softmax(dot, dim=2)
```





The following matrix denotes the values of $\frac{\mathbf{q}_i \cdot \mathbf{k}_j}{\sqrt{d_k}}$ for $1 \le i \le n, 1 \le j \le n \ (n=4)$

1	0	-1	-1
1	1	-1	0
0	1	1	-1
-1	-1	2	1

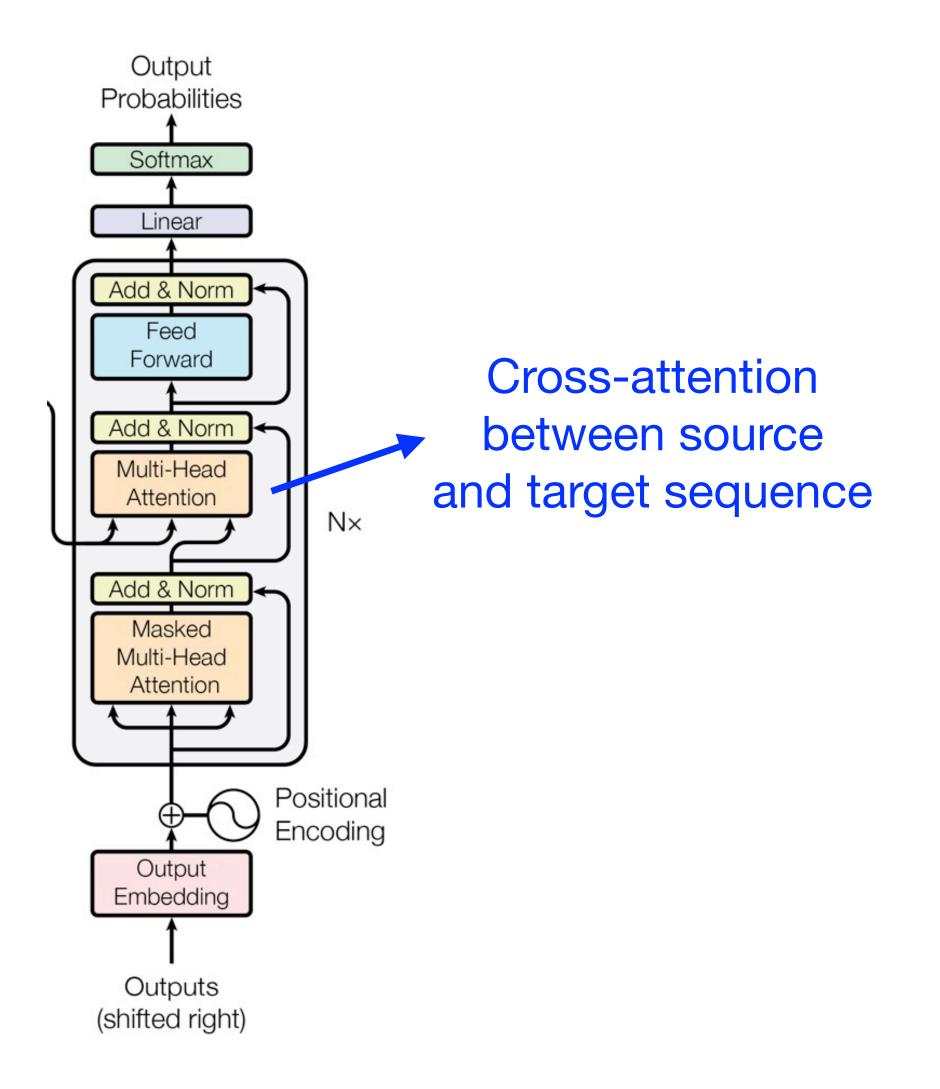
What should be the value of $\alpha_{2,2}$ in masked attention?

(A) 0
(B) 0.5
(C)
$$\frac{e}{2e + e^{-1} + 1}$$

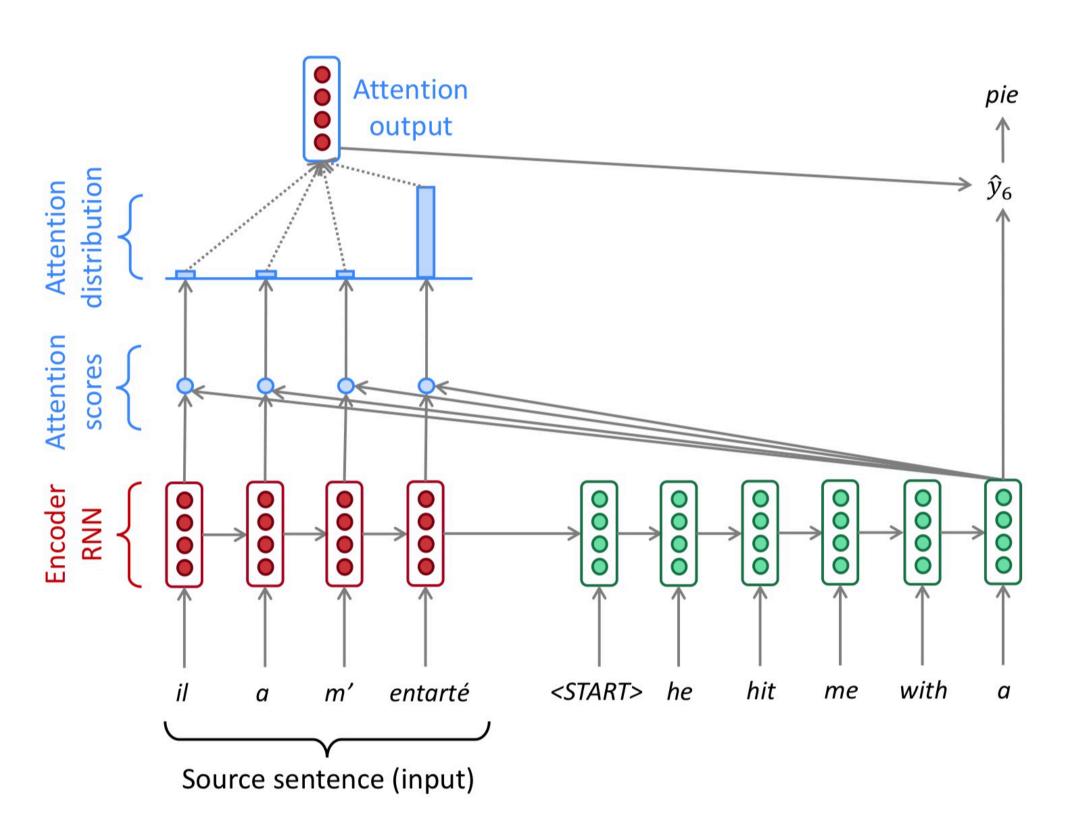
(D) 1

The correct answer is (B)

Multi-head cross-attention



Similar as the attention in seq2seq model!



Multi-head cross-attention

Self-attention:

$$\mathbf{q}_{i} = \mathbf{x}_{i} \mathbf{W}^{Q}, \mathbf{k}_{i} = \mathbf{x}_{i} \mathbf{W}^{K}, \mathbf{v}_{i} = \mathbf{x}_{i} \mathbf{W}^{V}$$

$$e_{i,j} = \frac{\mathbf{q}_{i} \cdot \mathbf{k}_{j}}{\sqrt{d_{k}}}, \forall j = 1, \dots, n$$

$$\alpha_{i} = \operatorname{softmax}(\mathbf{e}_{i})$$

$$\mathbf{h}_{i} = \sum_{i=1}^{n} \alpha_{i,j} \mathbf{v}_{j}$$

Cross-attention:

(always from the top layer)

 $ilde{\mathbf{x}}_1,\dots, ilde{\mathbf{x}}_m$: hidden states from encoder $\mathbf{x}_1,\dots,\mathbf{x}_n$: hidden states from decoder

$$\mathbf{q}_i = \mathbf{x}_i \mathbf{W}^Q \quad i = 1, 2, \dots, n$$

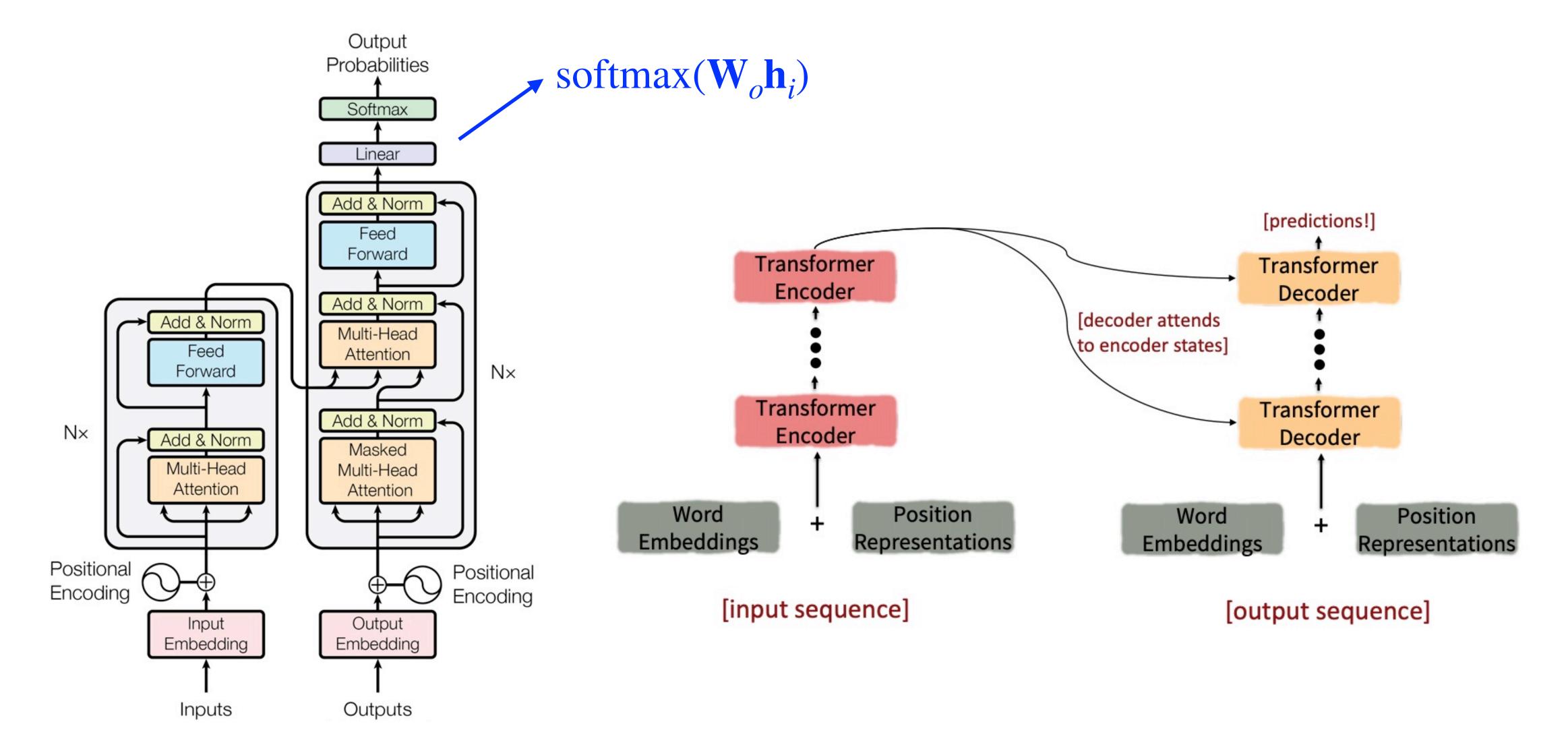
$$\mathbf{k}_j = \tilde{\mathbf{x}}_j \mathbf{W}^K, \mathbf{v}_j = \tilde{\mathbf{x}}_j \mathbf{W}^V \quad \forall j = 1, 2, ..., m$$

$$e_{i,j} = \frac{\mathbf{q_i} \cdot \mathbf{k}_j}{\sqrt{d_k}}, \forall j = 1, \dots, m$$

$$\alpha_i = \operatorname{softmax}(\mathbf{e}_i)$$

$$\mathbf{h}_i = \sum_{j=1}^m \alpha_{i,j} \mathbf{v}_j$$

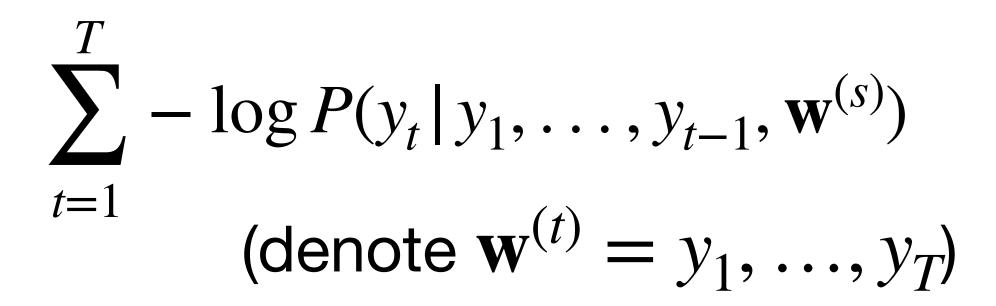
Transformer encoder-decoder

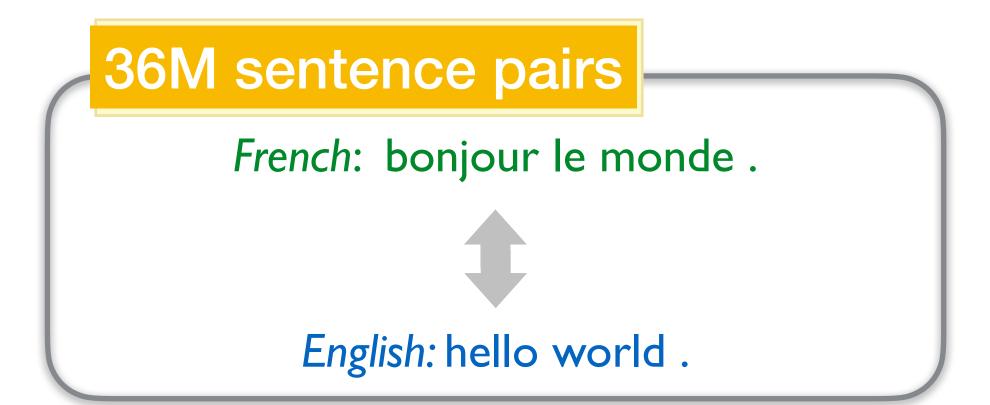


Training Transformer encoder-decoder models

The same as the way that we train seq2seq models!

- Training data: parallel corpus $\{(\mathbf{w}_i^{(s)}, \mathbf{w}_i^{(t)})\}$
- Minimize cross-entropy loss:





Back-propagate gradients through both encoder and decoder

Masked self-attention is the key!

This can enable parallelizable operations while NOT looking at the future

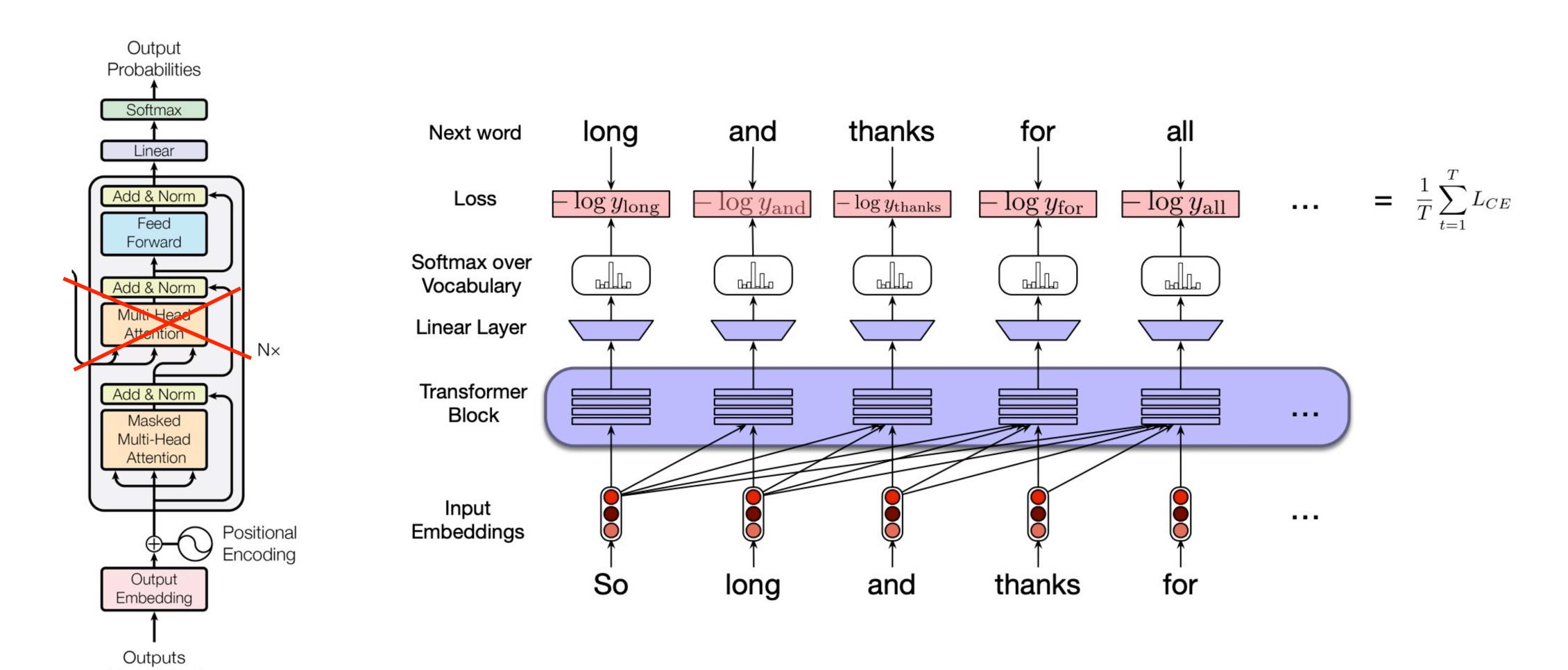
Empirical results with Transformers

Madal	BL	EU	Training Co	Training Cost (FLOPs)			
Model	EN-DE	EN-FR	EN-DE	EN-FR			
ByteNet [15]	23.75						
Deep-Att + PosUnk [32]		39.2		$1.0 \cdot 10^{20}$			
GNMT + RL [31]	24.6	39.92	$2.3\cdot 10^{19}$	$1.4\cdot 10^{20}$			
ConvS2S [8]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5\cdot 10^{20}$			
MoE [26]	26.03	40.56	$2.0\cdot 10^{19}$	$1.2\cdot 10^{20}$			
Deep-Att + PosUnk Ensemble [32]		40.4		$8.0 \cdot 10^{20}$			
GNMT + RL Ensemble [31]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1\cdot 10^{21}$			
ConvS2S Ensemble [8]	26.36	41.29	$7.7\cdot 10^{19}$	$1.2\cdot 10^{21}$			
Transformer (base model)	27.3	38.1	3.3 ·	10 ¹⁸			
Transformer (big)	28.4	41.0	$2.3 \cdot$	10^{19}			

(Vaswani et al., 2017)

Transformer-based language models

The backbone of large language models (e.g., GPT/ChatGPT, Gemini, LLaMA, ...)

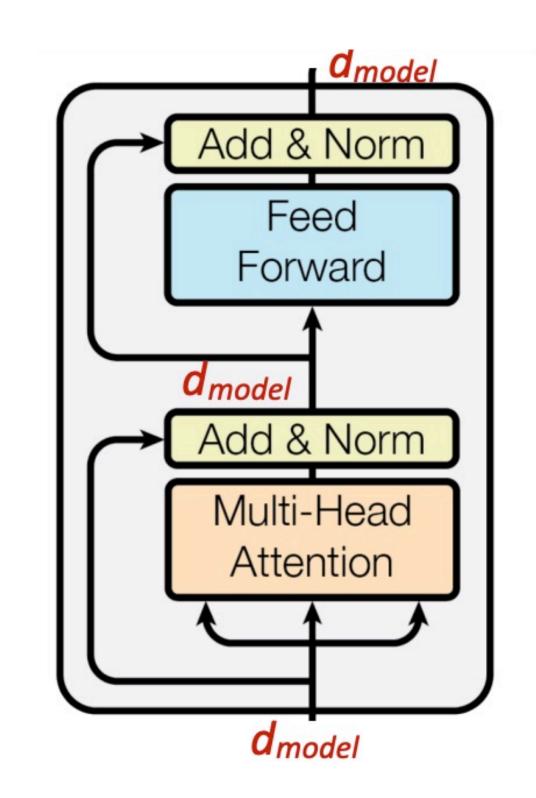


Transformer architecture specifications

	N	$d_{ m model}$	$d_{ m ff}$	h	d_k	d_v
base	6	512	2048	8	64	64

(Vaswani et al., 2017)

Model Name	$n_{ m params}$	$n_{ m layers}$	$d_{ m model}$	$n_{ m heads}$	$d_{ m head}$
GPT-3 Small	125M	12	768	12	64
GPT-3 Medium	350M	24	1024	16	64
GPT-3 Large	760M	24	1536	16	96
GPT-3 XL	1.3B	24	2048	24	128
GPT-3 2.7B	2.7B	32	2560	32	80
GPT-3 6.7B	6.7B	32	4096	32	128
GPT-3 13B	13.0B	40	5140	40	128
GPT-3 175B or "GPT-3"	175.0B	96	12288	96	128



(Brown et al., 2020)

Transformers: pros and cons

- Easier to capture long-range dependencies: we draw attention between every pair of words!
- Easier to parallelize:

$$Q = XW^{Q} \quad K = XW^{K} \quad V = XW^{V}$$
 Attention $(Q, K, V) = \operatorname{softmax}(\frac{QK^{T}}{\sqrt{d_{k}}})V$

Are positional embeddings enough to capture positional information?

Otherwise self-attention is an unordered function of its input

Quadratic computation in self-attention

Can become very slow when the sequence becomes very long

Computational analysis of Transformers

Multi-head attention (MHA)

$$Q = XW^{Q}, K = XW^{K}, V = XW^{V}$$

$$n \times d \qquad d \times n$$

$$Attention(Q, K, V) = \operatorname{softmax}(\frac{QK^{T}}{\sqrt{d_{k}}})V \qquad n \times d \qquad O(nd^{2} + n^{2}d)$$

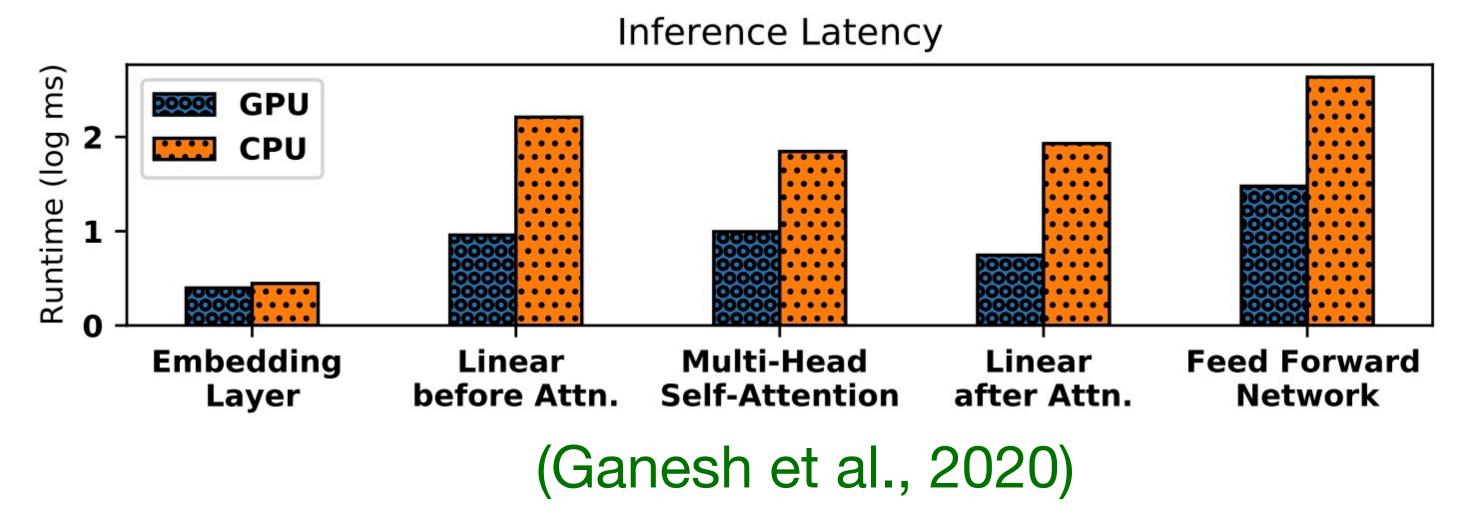
Feed-forward layers (FFN)

$$d imes d_{ff} imes d_{ff}$$

Note: RNNs only require $O(nd^2)$ time: $\mathbf{h}_t = f(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_t + \mathbf{b})$ (assuming input dimension = hidden dimension = d)

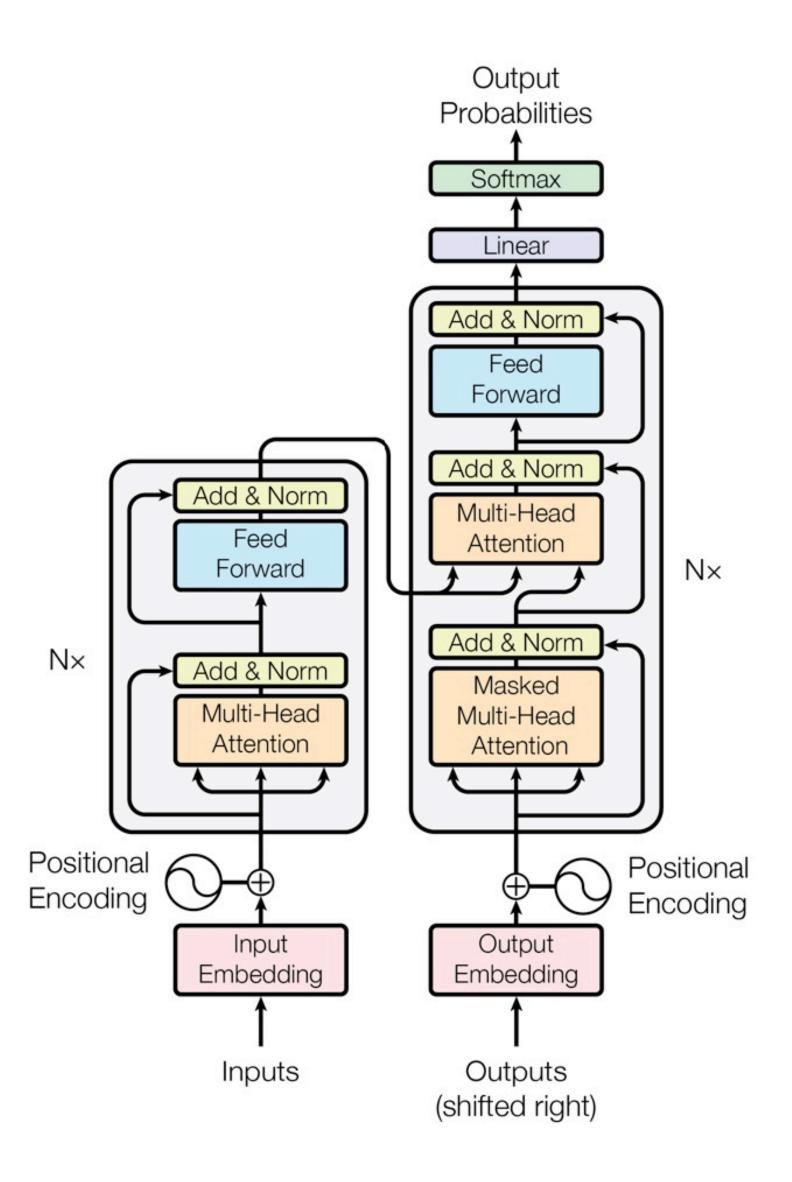
Computational analysis of Transformers

• For BERT-sized models (n=512, d=768, $d_{ff}=4d$), 2/3 of parameters are FFNs.



- However, when sequence length becomes longer (e.g., > 50,000), the computation will be dominated by self-attention $O(n^2d)$
 - Numerous solutions have been proposed to address this issue
 - Long-context language modeling is still one of the most active research areas today

Transformers: roadmap



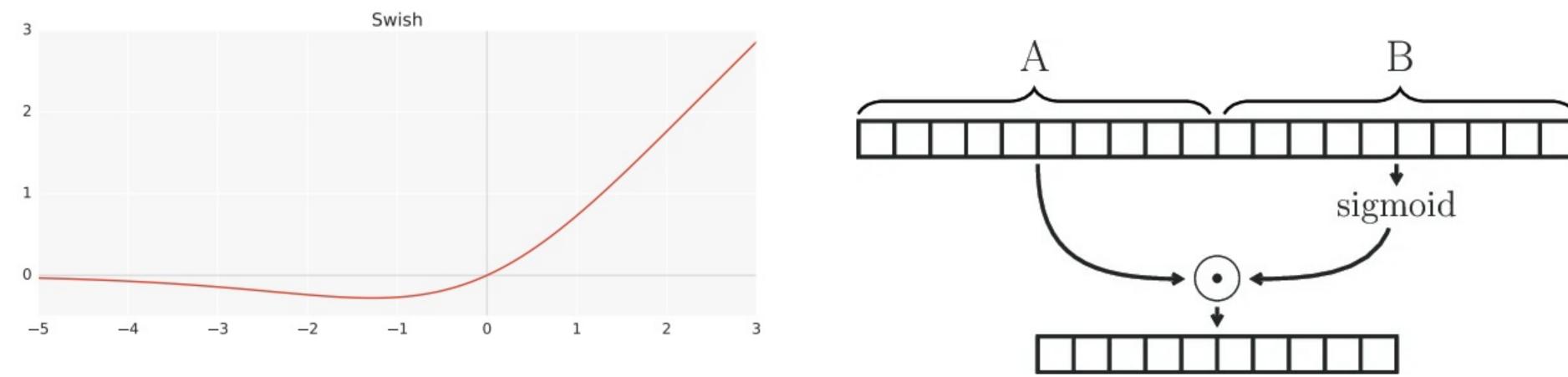
- Self-attention and multi-head attention
- Feedforward layers
- Positional encoding
- Residual connections + layer normalization
- Transformer encoder vs Transformer decoder
- Advanced techniques: SwiGLU, rotary embeddings, pre-normalization, grouped query attention
- Architecture exploration beyond Transformers

Major modifications since original Transformers

SwiGLU activation

SwiGLU = Swish + GLU

$$\text{SwiGLU}(x, W, V, b, c, \beta) = \text{Swish}_{\beta}(xW + b) \otimes (xV + c)$$



 $Swish(x) = x \cdot sigmoid(\beta x)$

https://azizbelaweid.substack.com/p/what-is-swiglu-how-to-implement-it

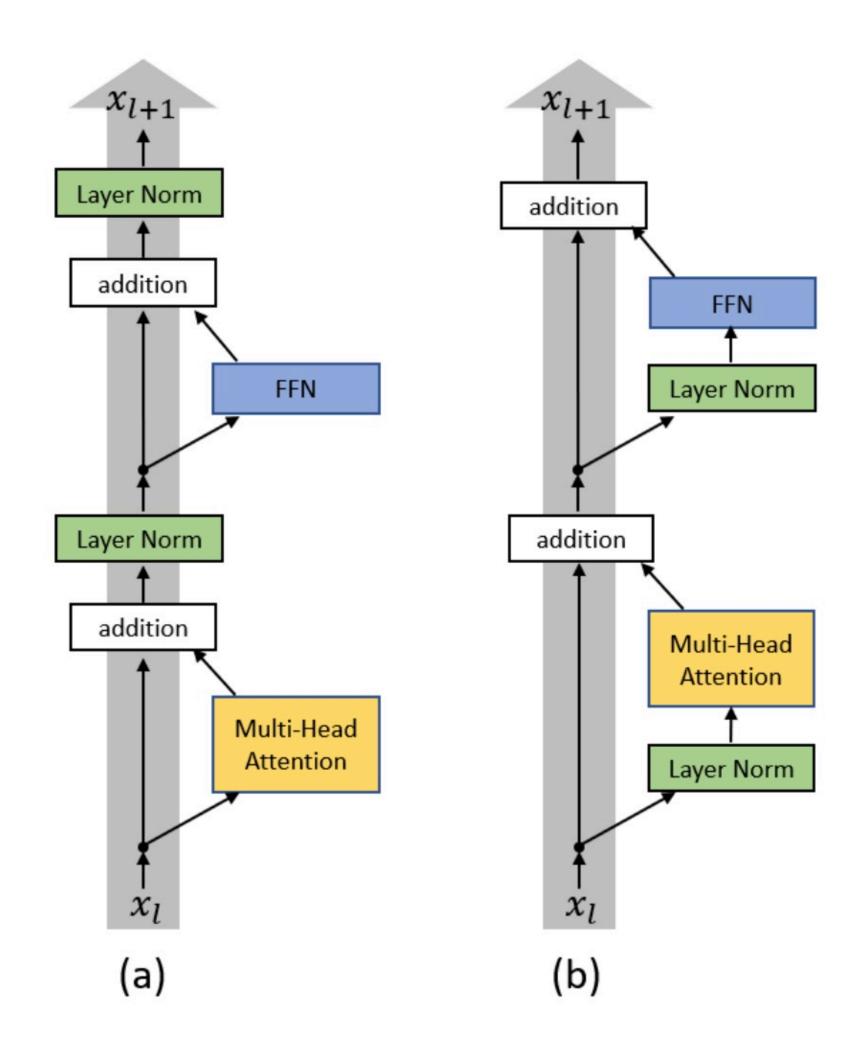
SwiGLU activation

	Score	CoLA	SST-2	MRPC	MRPC	STSB	STSB	QQP	QQP	MNLIm	MNLImm	QNLI	RTE
	Average	MCC	Acc	F1	Acc	PCC	SCC	F1	Acc	Acc	Acc	Acc	Acc
$\mathrm{FFN}_{\mathrm{ReLU}}$	83.80	51.32	94.04	93.08	90.20	89.64	89.42	89.01	91.75	85.83	86.42	92.81	80.14
$\mathrm{FFN}_{\mathrm{GELU}}$	83.86	53.48	94.04	92.81	90.20	89.69	89.49	88.63	91.62	85.89	86.13	92.39	80.51
$\mathrm{FFN}_{\mathrm{Swish}}$	83.60	49.79	93.69	92.31	89.46	89.20	88.98	88.84	91.67	85.22	85.02	92.33	81.23
$\mathrm{FFN}_{\mathrm{GLU}}$	84.20	49.16	94.27	92.39	89.46	89.46	89.35	88.79	91.62	86.36	86.18	92.92	84.12
FFN_{GEGLU}	84.12	53.65	93.92	92.68	89.71	90.26	90.13	89.11	91.85	86.15	86.17	92.81	79.42
$FFN_{Bilinear}$	83.79	51.02	94.38	92.28	89.46	90.06	89.84	88.95	91.69	86.90	87.08	92.92	81.95
FFN_{SwiGLU}	84.36	51.59	93.92	92.23	88.97	90.32	90.13	89.14	91.87	86.45	86.47	92.93	83.39
$\mathrm{FFN}_{\mathrm{ReGLU}}$	84.67	56.16	94.38	92.06	89.22	89.97	89.85	88.86	91.72	86.20	86.40	92.68	81.59
[Raffel et al., 2019]	83.28	53.84	92.68	92.07	88.92	88.02	87.94	88.67	91.56	84.24	84.57	90.48	76.28
ibid. stddev.	0.235	1.111	0.569	0.729	1.019	0.374	0.418	0.108	0.070	0.291	0.231	0.361	1.393

$$\text{SwiGLU}(x, W, V, b, c, \beta) = \text{Swish}_{\beta}(xW + b) \otimes (xV + c)$$

Notes: there are 3 projection matrices (up_project, down_project, gate_project), d_{ff} is reduced to $4d \times \frac{2}{3}$

Pre-normalization



RMSNorm normalization function

$$ar{a}_i = rac{a_i}{ ext{RMS}(\mathbf{a})} g_i, \quad ext{where } ext{RMS}(\mathbf{a}) = \sqrt{rac{1}{n} \sum_{i=1}^n a_i^2}.$$

(Zhang and Senrich, 2019)

Image: (Xiong et al., 2020)

Rotary positional embeddings

Relative positional embeddings (T5 uses this!):

Self-Attention with Relative Position Representations

Peter Shaw
Google
petershaw@google.com

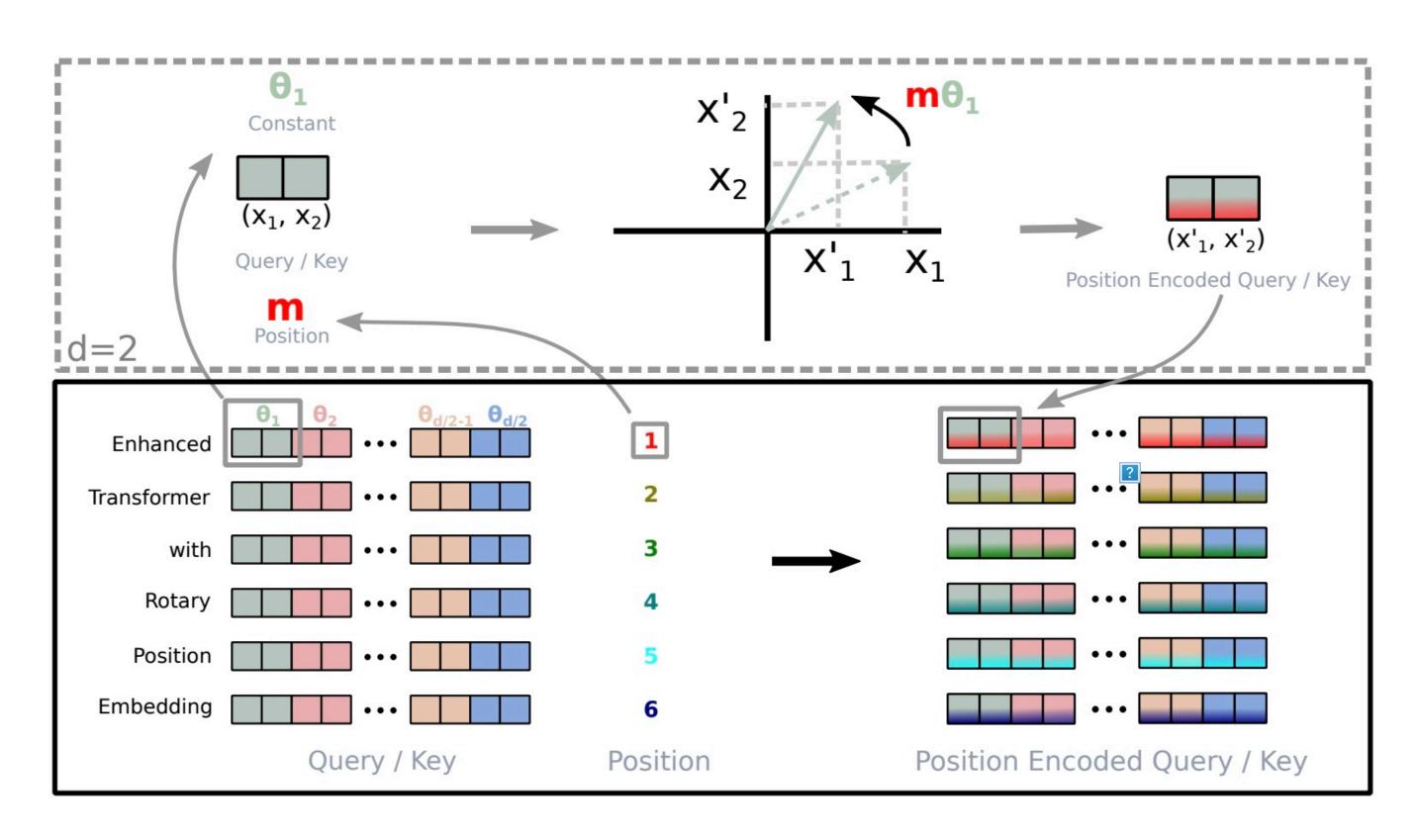
Jakob Uszkoreit Google Brain usz@google.com

Ashish Vaswani Google Brain avaswani@google.com

$$e_{ij} = \frac{x_i W^Q (x_j W^K + \mathbf{a}_{ij}^K)^T}{\sqrt{d_z}}$$

- Instead of focusing on absolute positions, relative positional embeddings concentrate on the **distances between pairs of tokens**
- Incorporating this relative positional information into attention directly

Rotary positional embeddings

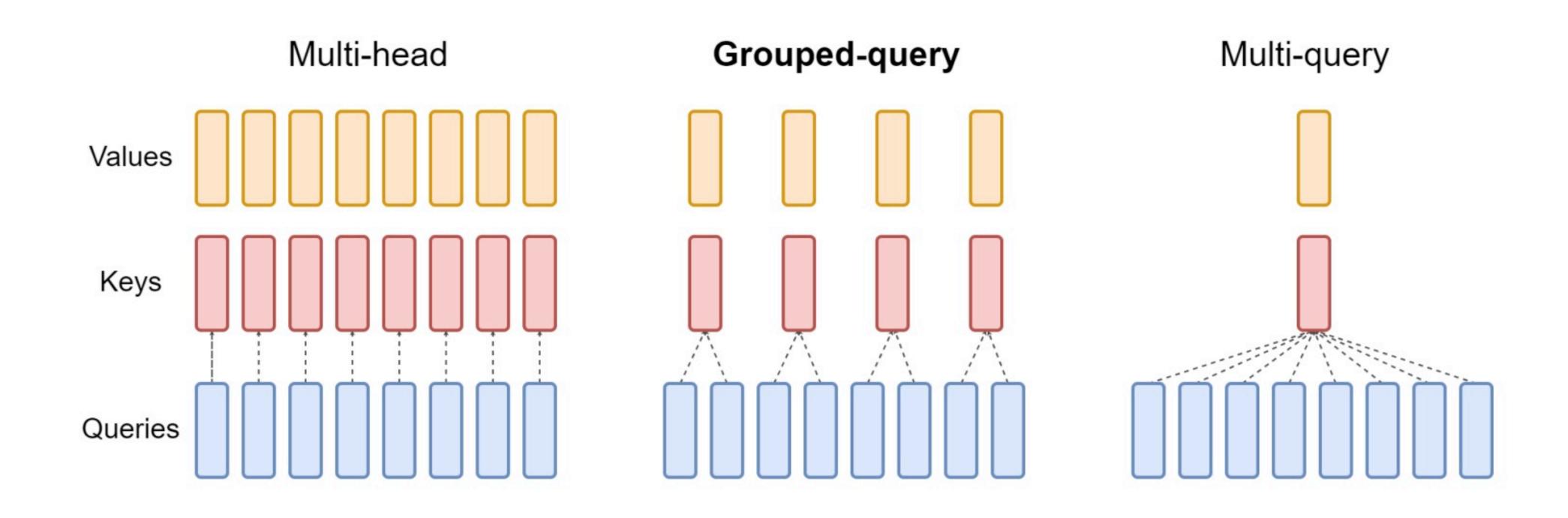


Unites both **absolute** and **relative** positional information

$$f_{\{q,k\}}(\boldsymbol{x}_{m},m) = \begin{pmatrix} \cos m\theta & -\sin m\theta \\ \sin m\theta & \cos m\theta \end{pmatrix} \begin{pmatrix} W_{\{q,k\}}^{(11)} & W_{\{q,k\}}^{(12)} \\ W_{\{q,k\}}^{(21)} & W_{\{q,k\}}^{(22)} \end{pmatrix} \begin{pmatrix} x_{m}^{(1)} \\ x_{m}^{(2)} \end{pmatrix}$$

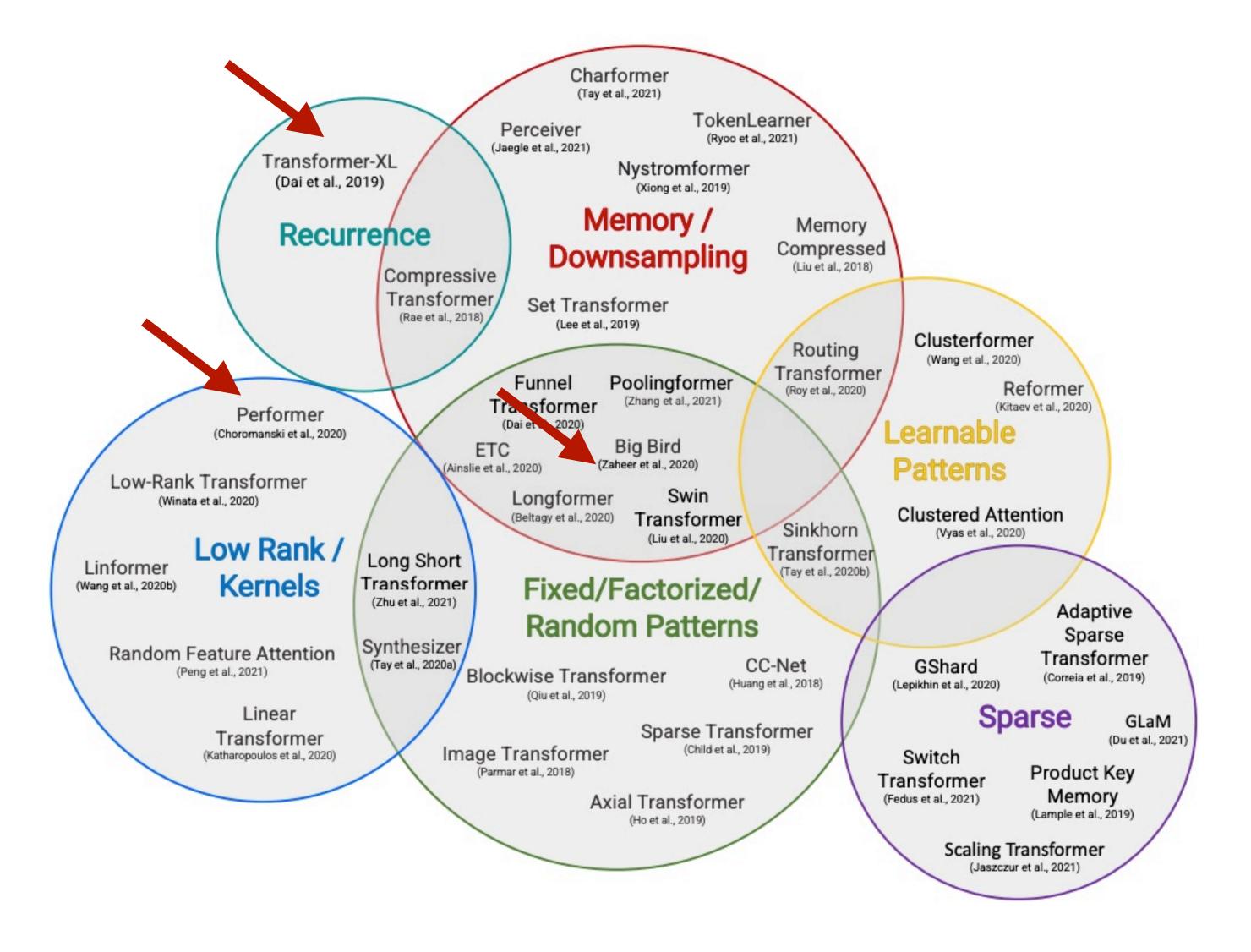
(Su et al., 2021) RoFormer: Enhanced Transformer with Rotary Position Embedding

Grouped query attention (GQA)

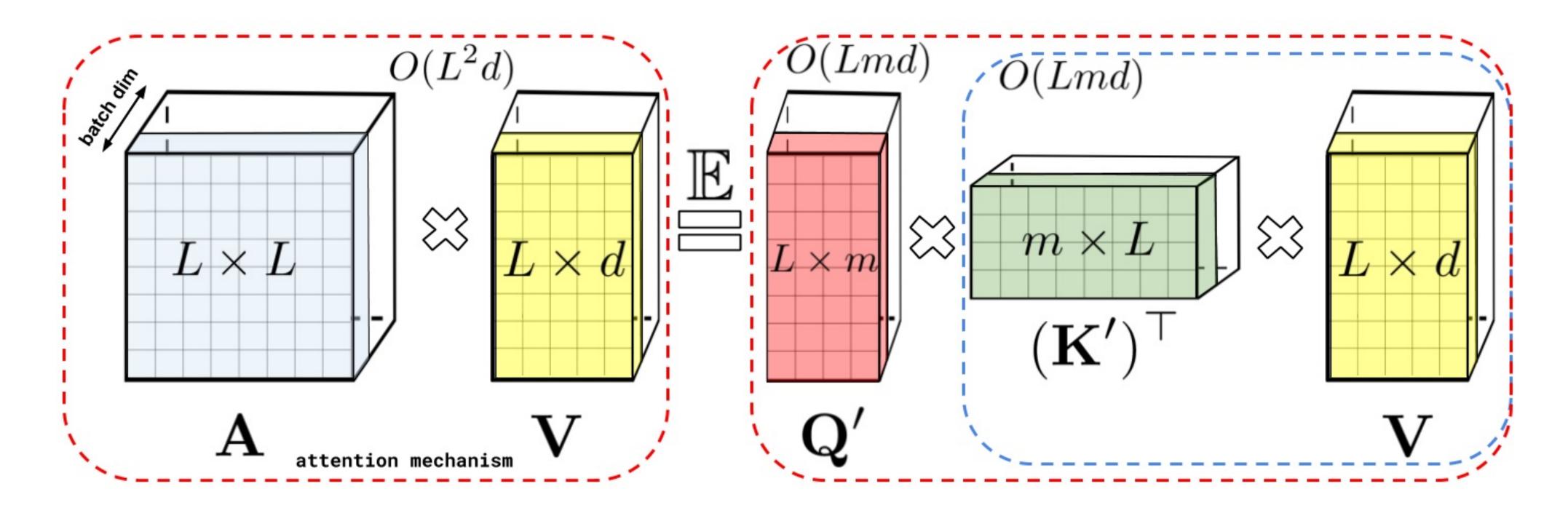


Architecture exploration beyond Transformers

Efficient Transformers



Example: Performers

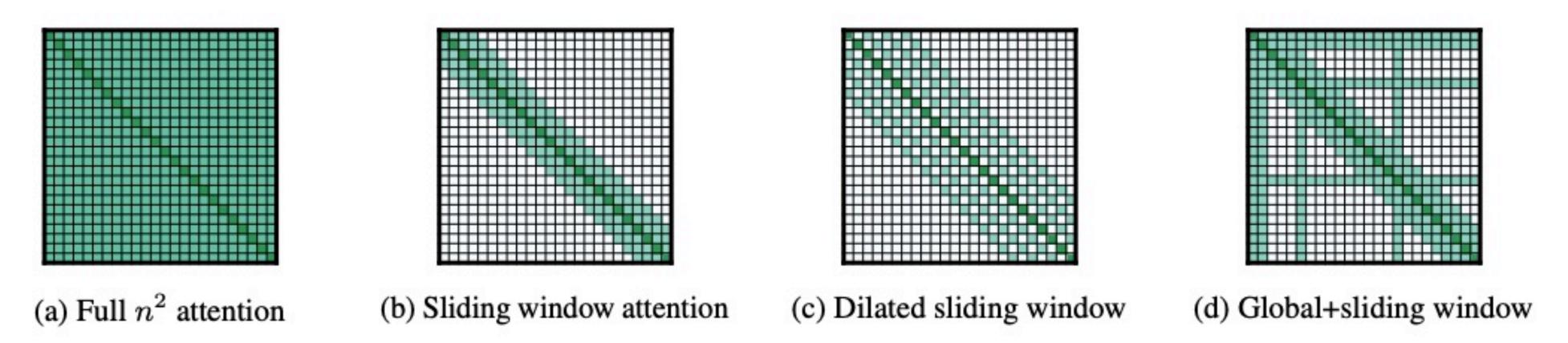


L: sequence length, m << L

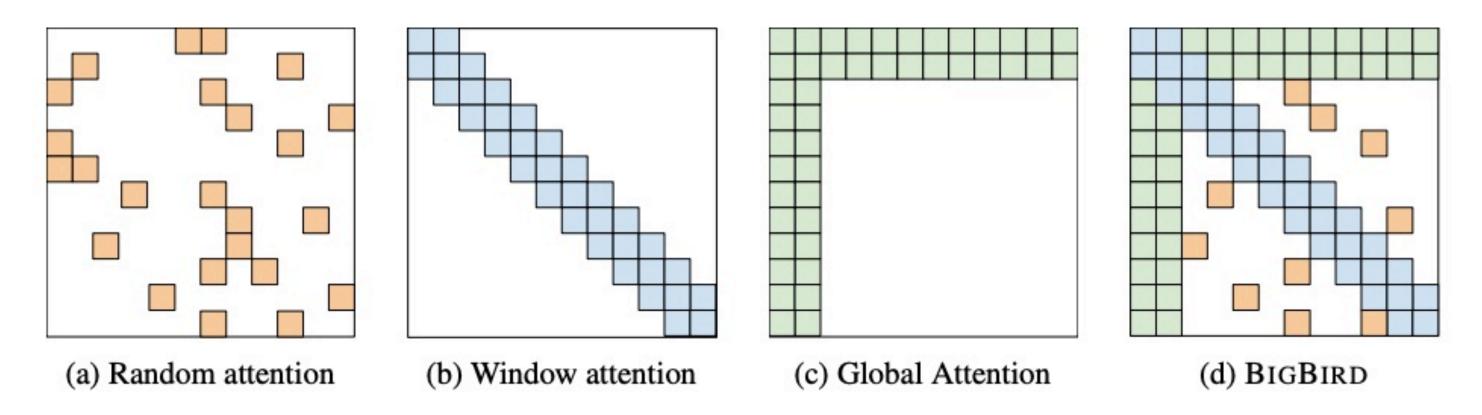
Low-rank decomposition: Decompose A as the product of Q' and K' (random projection of origional keys and querys)

Example: Longformer / Big Bird

Sparse attention: only compute attention at particular positions



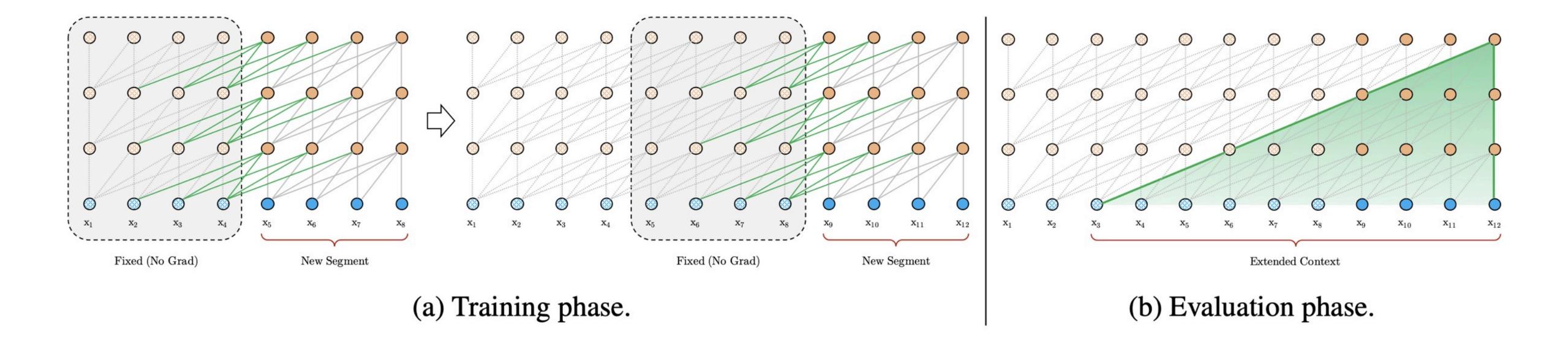
(Beltagy et al., 2020): Longformer: The Long-Document Transformer



(Zaheer et al., 2021): Big Bird: Transformers for Longer Sequences

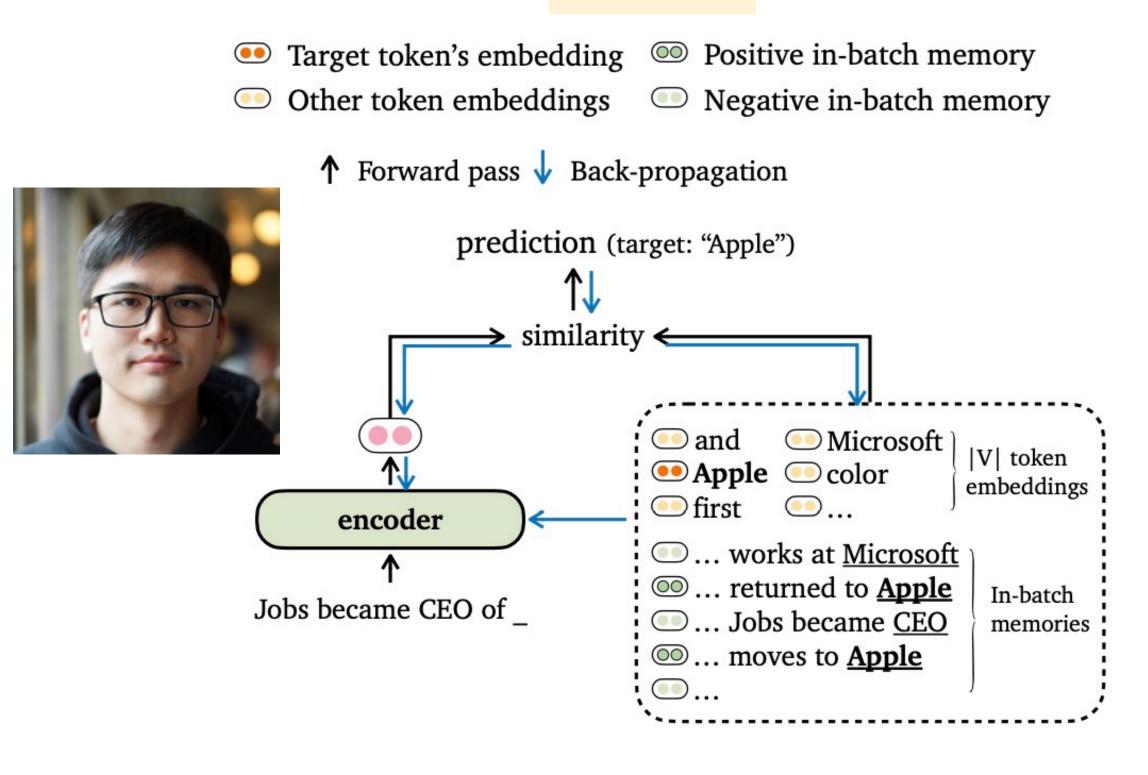
Example: Transformer-XL

Segment-level recurrence with state reuse: hidden representations from previous segment will be cached as extended context (no back-propagation to those!)



Research from my group

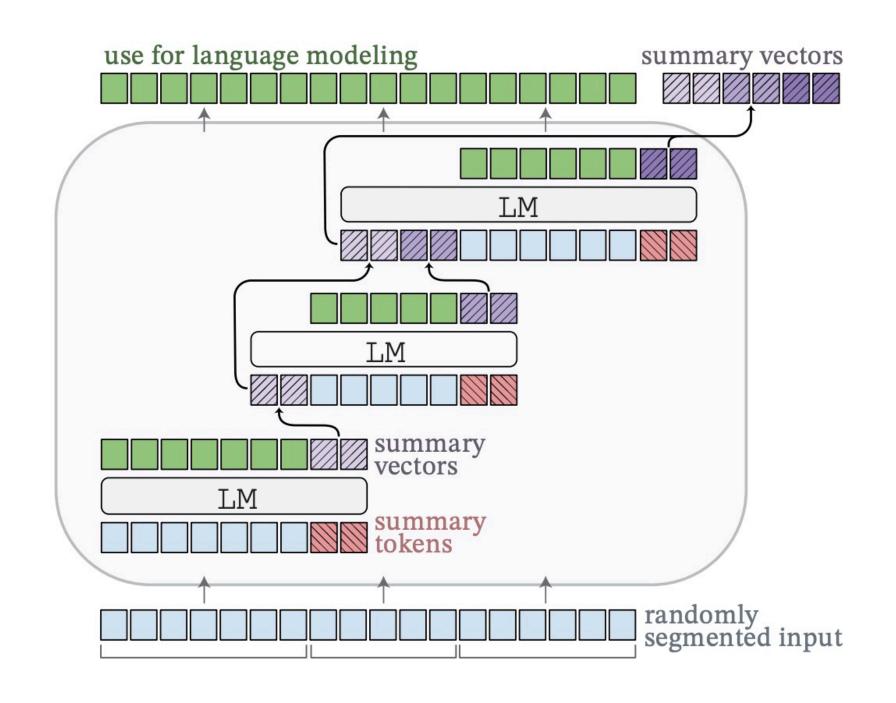
TRIME







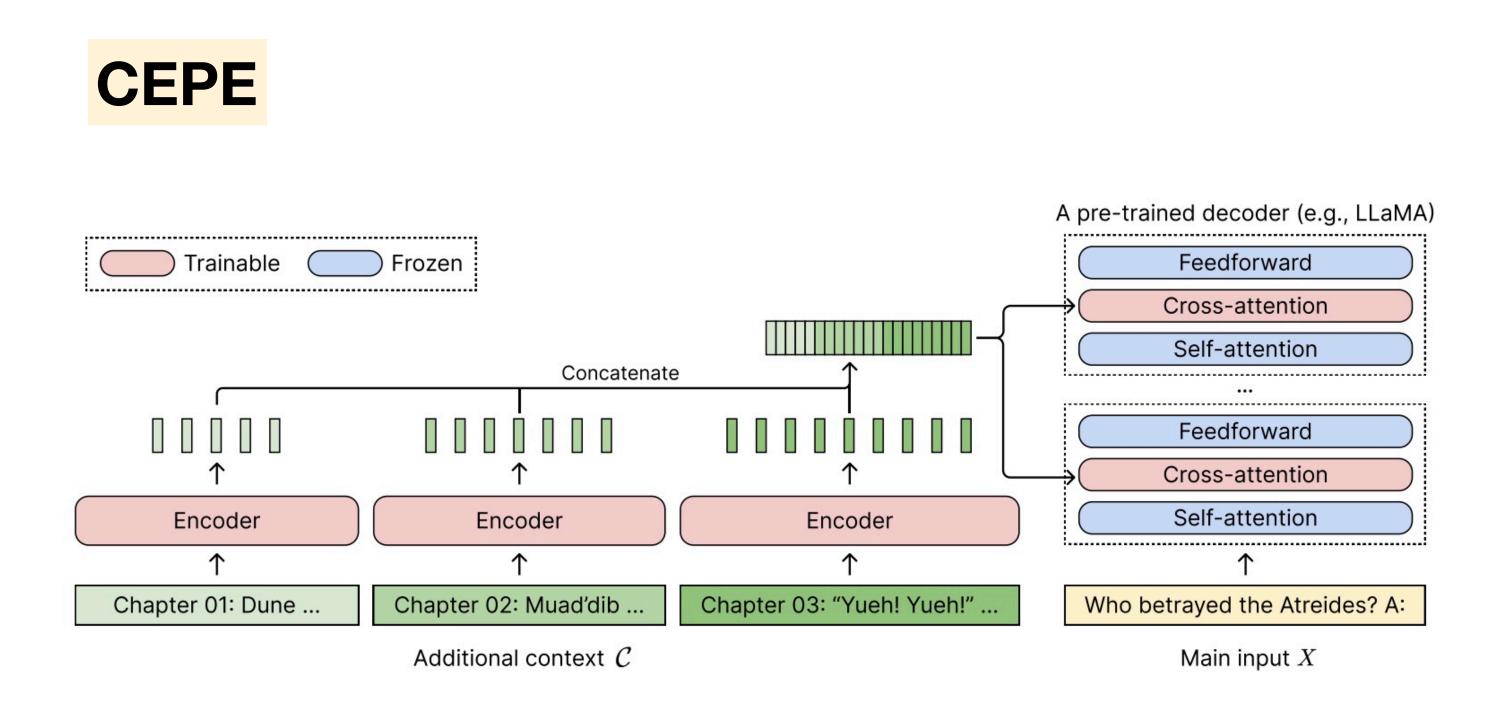
AutoCompressors



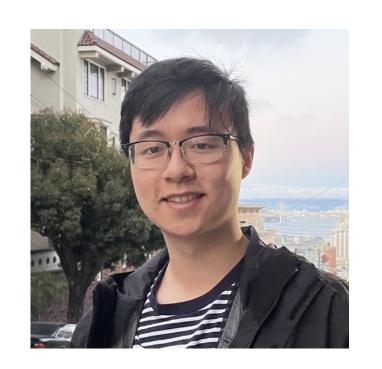
(Zhong et al., EMNLP'22) Training Language Models with Memory Augmentation

(Chevalier et al., EMNLP'23) Adapting Language Models to Compress Contexts

Research from my group





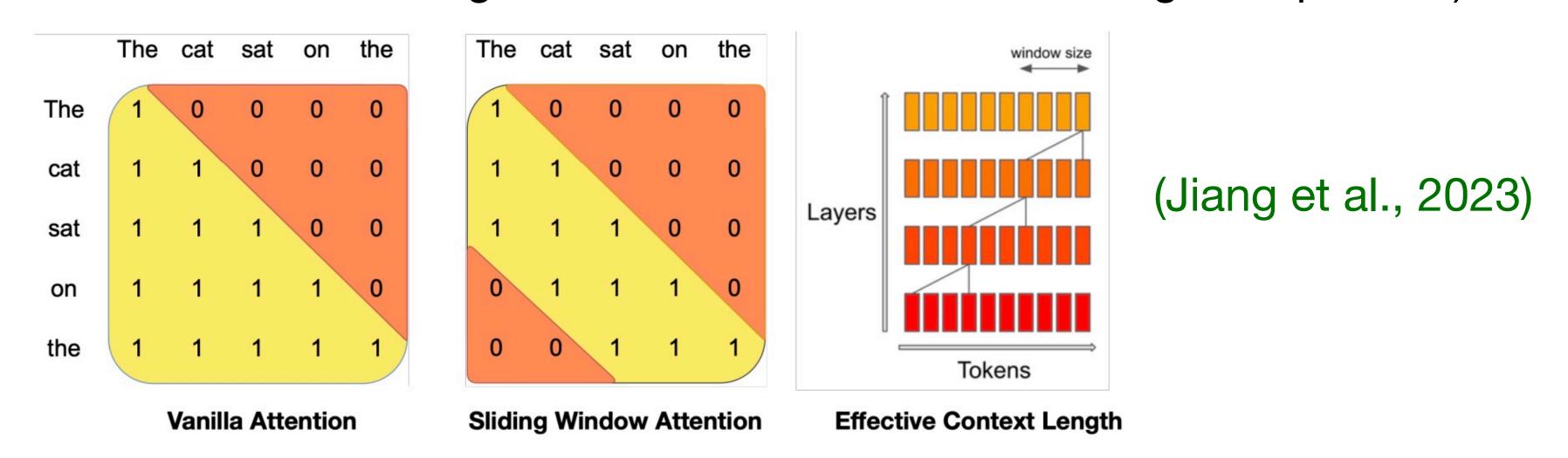




These architectures/techniques are generally applicable to both long-context modeling and retrieval augmentation!

Remarks on efficient Transformers

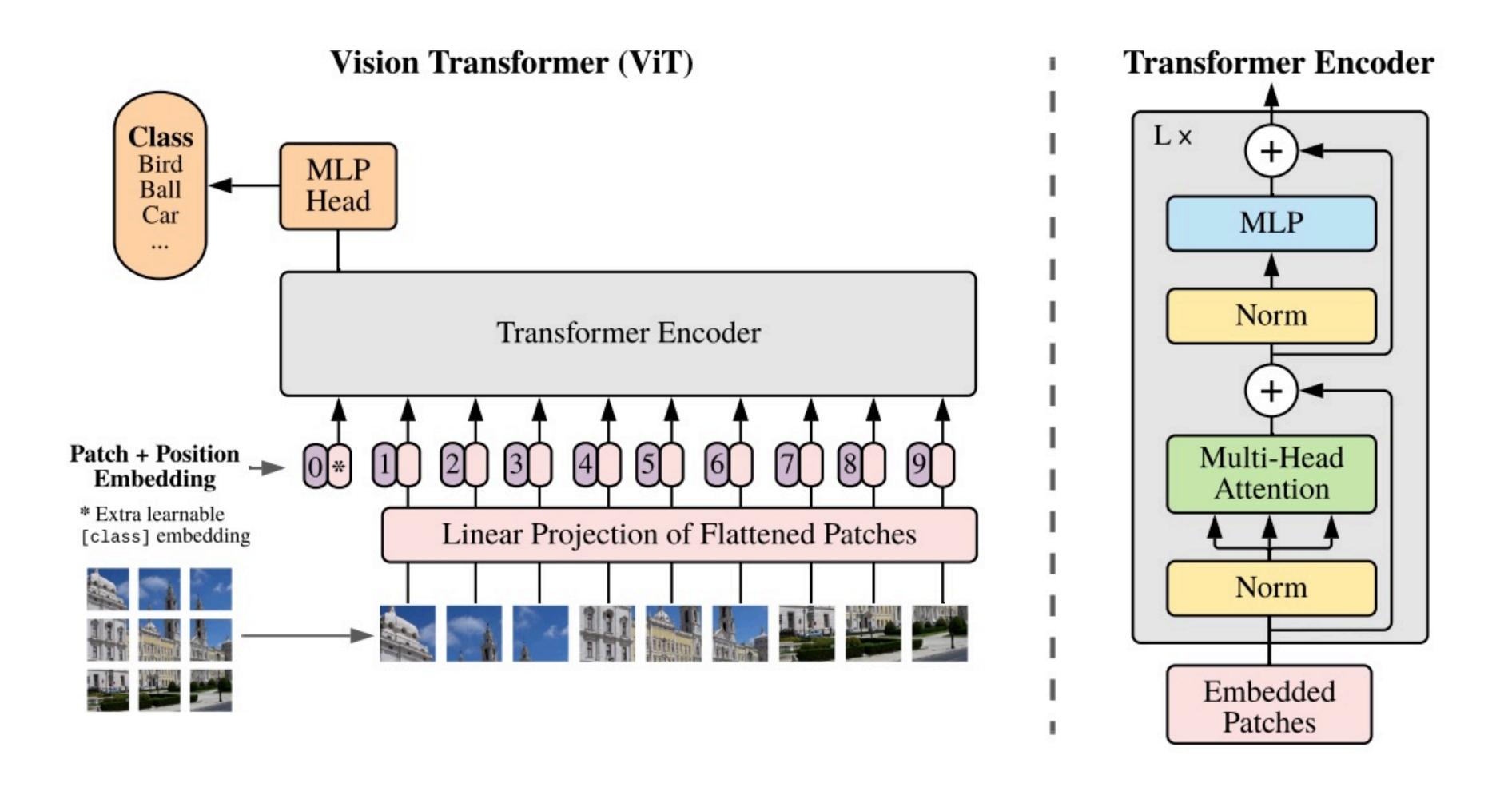
• A lot of exploration around 2019-2021: mostly approximation solutions of replacing the full quadratic attention. Few techniques have been adopted in state-of-the-art LLMs (exception: Mistral uses *Sliding Window Attention* to handle longer sequences).



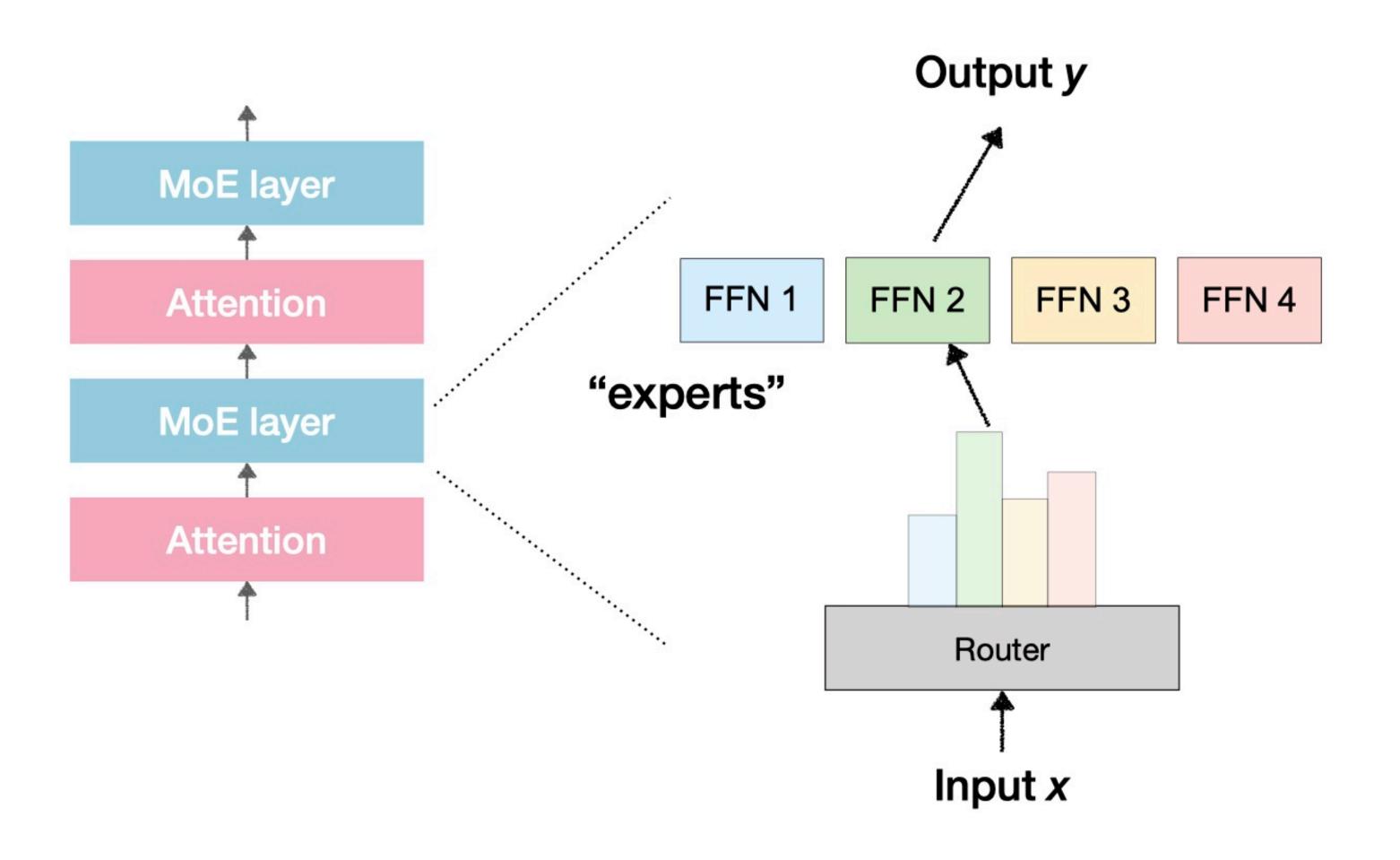
You can still do such approximations to speed up inference though!

• On the other hand, many system-level advancements have been made to scale up Transformers to longer sequences **without approximation** e.g., FlashAttention (Dao et al., 2022)

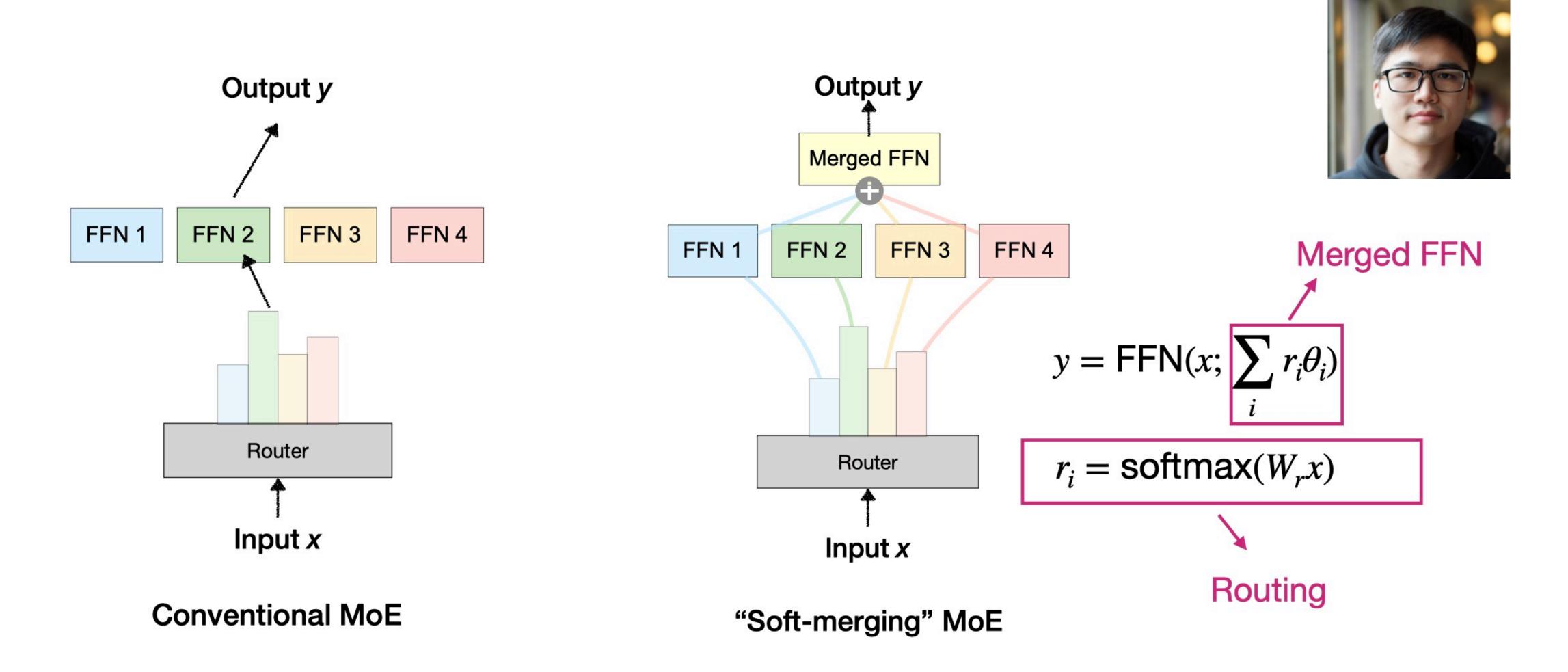
Vision Transformer (ViT)



Mixture of experts (MoEs)



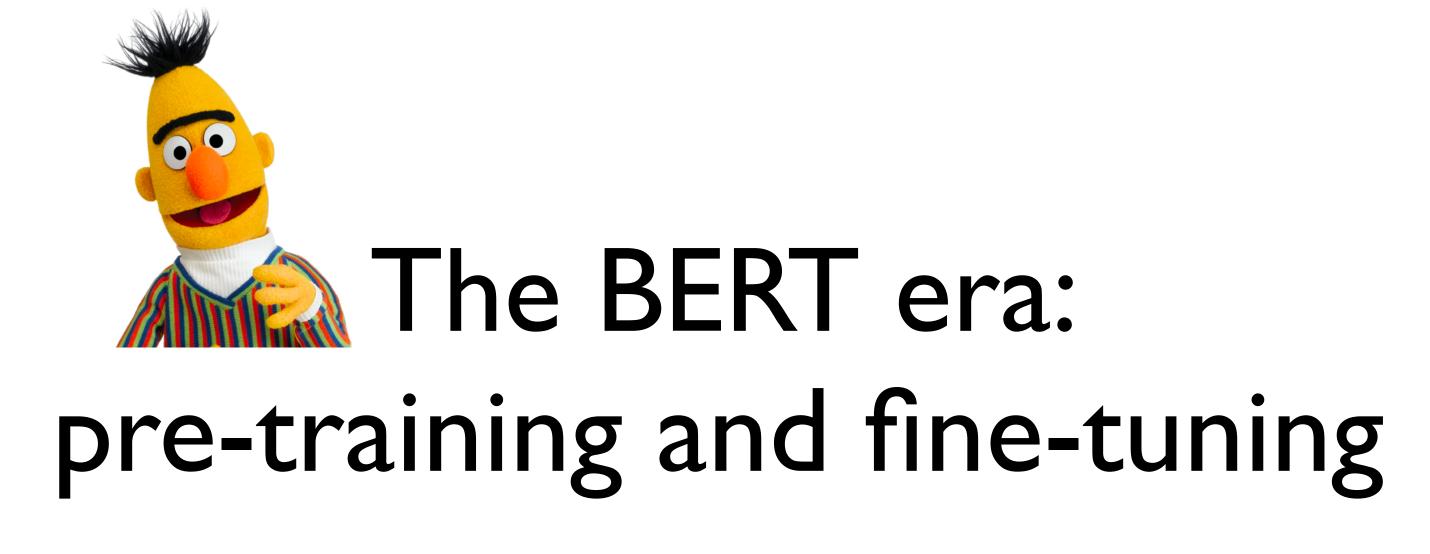
Mixture of experts (MoEs)



Part II. Pre-trained language models

Roadmap

- The BERT era: pre-training and fine-tuning
- The GPT-3 era: prompting and in-context learning
- The ChatGPT era: supervised instruction tuning and RLHF



BERT = Bidirectional Encoder Representations from Transformers

BERT: Pre-training of Deep Bidirectional Transformers for Language Understanding

Jacob Devlin Ming-Wei Chang Kenton Lee Kristina Toutanova
Google AI Language

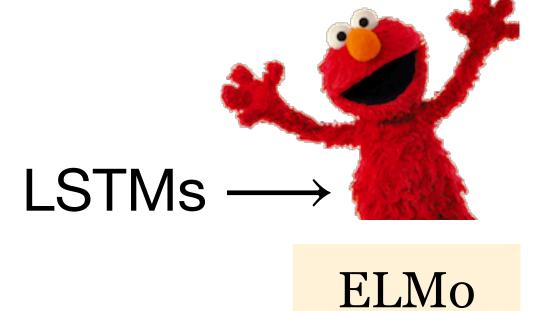
{jacobdevlin, mingweichang, kentonl, kristout}@google.com

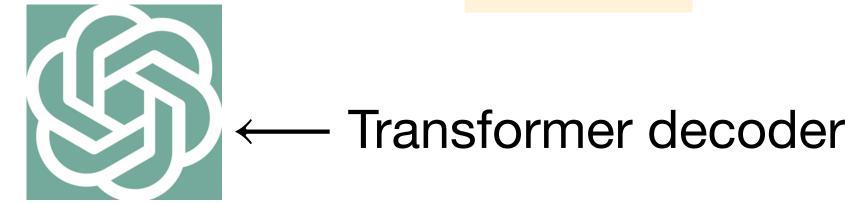
(Devlin et al., 2019)

Input: a sequence of *n* words

Output: a sequence of *n* vectors aka. "contextualized word embeddings"

Each word doesn't have a fixed vector as in (static) word embeddings



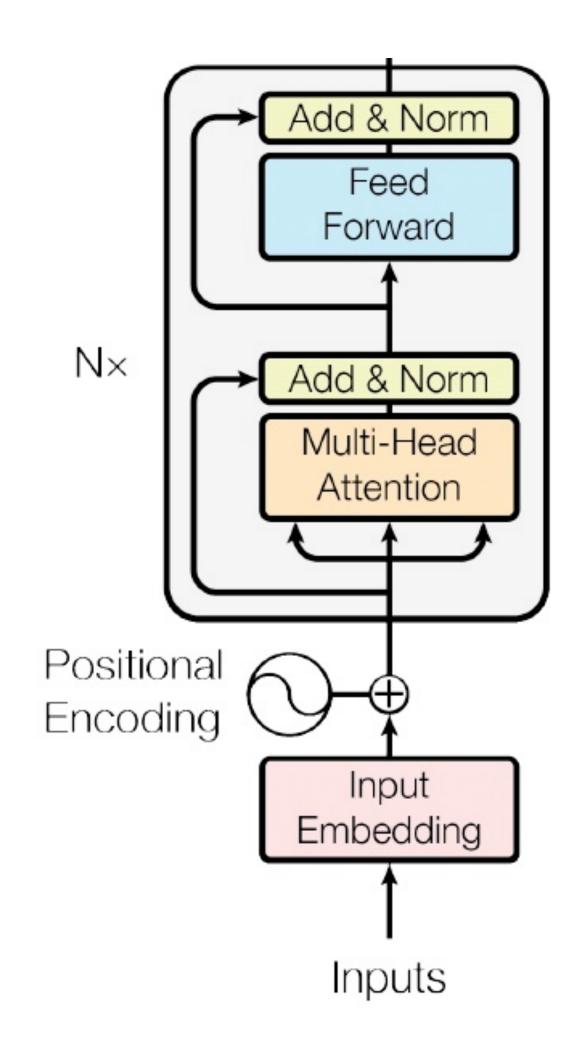


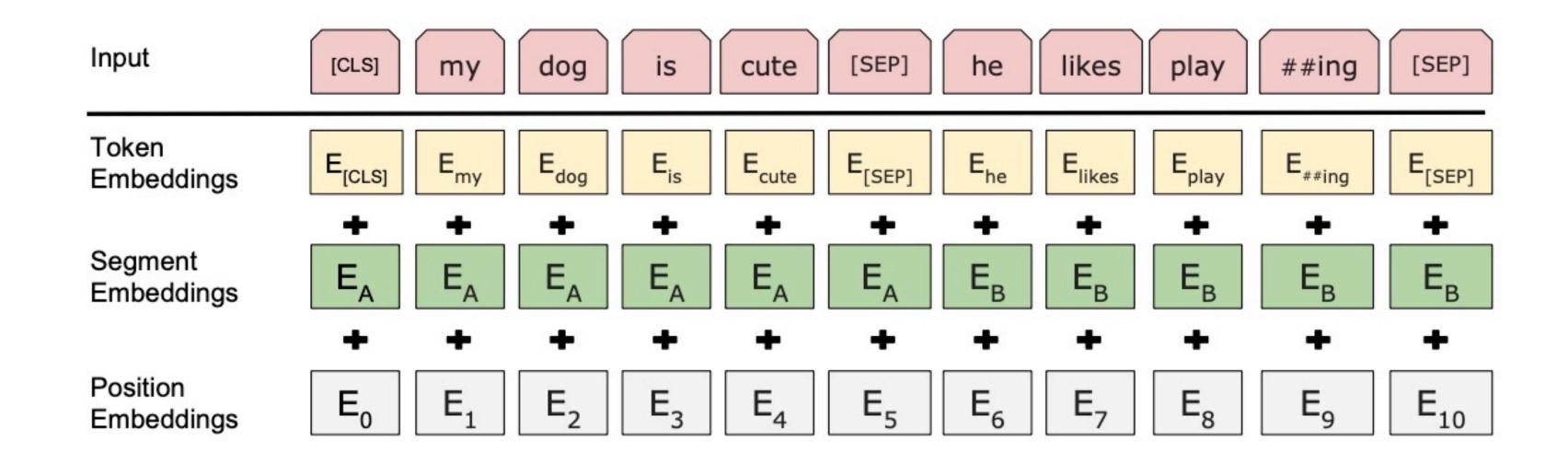




BERT

Inside BERT: A Transformer encoder





- 12 or 24 layers of Transformer blocks
- Each block consists of a multi-head self-attention layer and a feedforward layer with residual connections

How is BERT pre-trained?

Two pre-training objectives:

- Masked language modeling (MLM)
- Next sentence prediction (NSP)

Masked language modeling

= mask out 15% of the inputs and then predict the masked words

store gallon 个 个 the man went to the [MASK] to buy a [MASK] of milk

- Rather than always replacing the chosen words with [MASK], the data generator will do the following:
- 80% of the time: Replace the word with the [MASK] token, e.g., my dog is hairy → my dog is [MASK]
- 10% of the time: Replace the word with a random word, e.g., my dog is hairy → my dog is apple
- 10% of the time: Keep the word unchanged, e.g., my dog is hairy → my dog is hairy. The purpose of this is to bias the representation towards the actual observed word.
- Too little masking: too expensive to train
- Too much masking: not enough context

How is BERT pre-trained?

Two pre-training objectives:

- Masked language modeling (MLM)
- Next sentence prediction (NSP)

Next sentence prediction

Sample two segments of text (segment A and segment B) and predict whether the second segment is followed after the first one.

- 50% probability: a text segment of 512 tokens
- 50% probability: a text segment of 256 tokens, followed by another text segment of 256 tokens from a different document

The MLM loss and NSP loss are combined in pre-training

Remarks on BERT's pre-training objectives

- Later on, (Joshi et al., 2019; Liu et al., 2019) find that the next-sentence prediction objective unnecessary RoBERTa doesn't use NSP at all.
- Understanding the role of masking rates (Why 15%?)





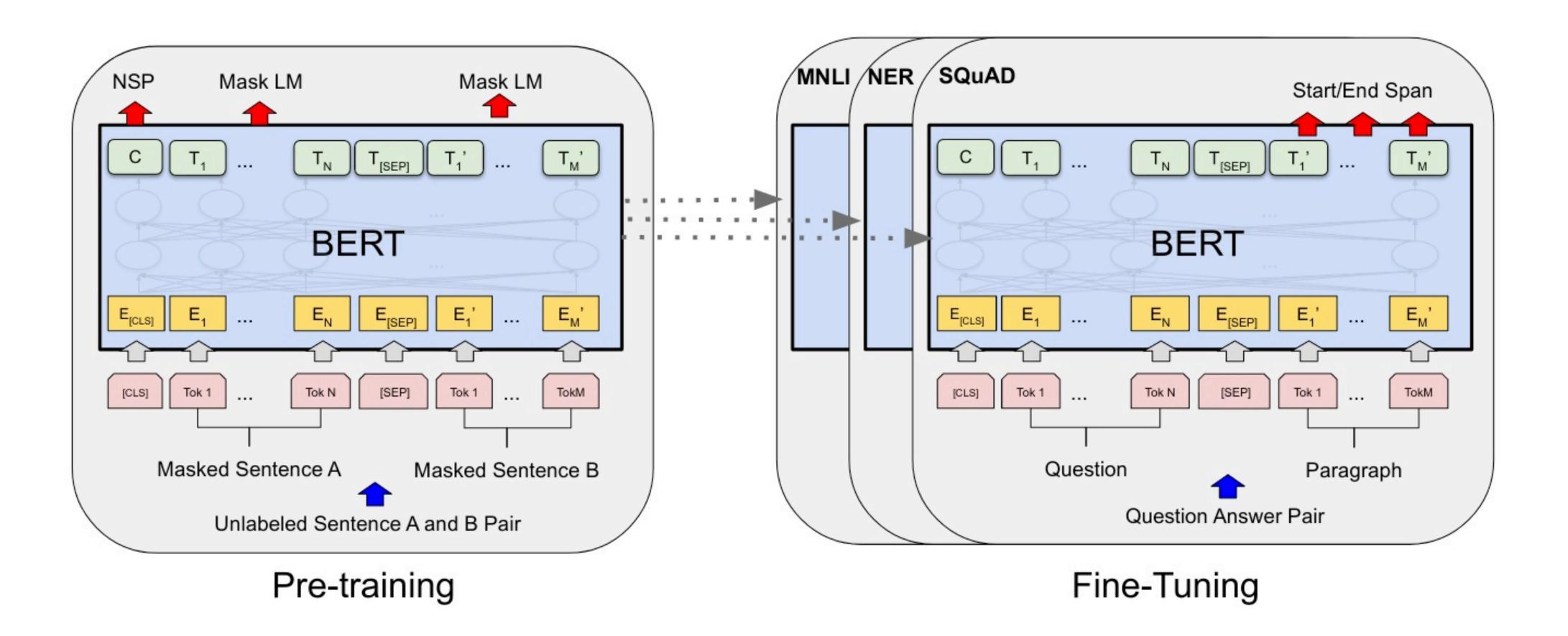
Should You Mask 15% in Masked Language Modeling?

Alexander Wettig* Tianyu Gao* Zexuan Zhong Danqi Chen
Department of Computer Science, Princeton University
{awettig, tianyug, zzhong, danqic}@cs.princeton.edu

(Wetting et al., 2023)

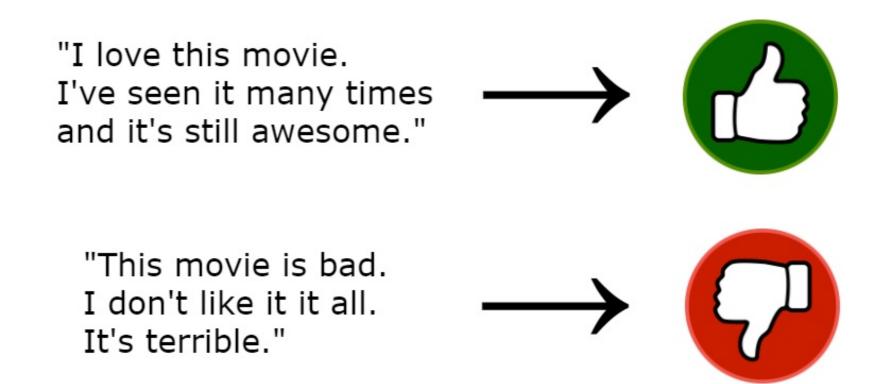
- The optimal masking rate should depend on model sizes and masking strategies
- Masking plays two distinct roles: corruption vs prediction

How is BERT used for downstream tasks?

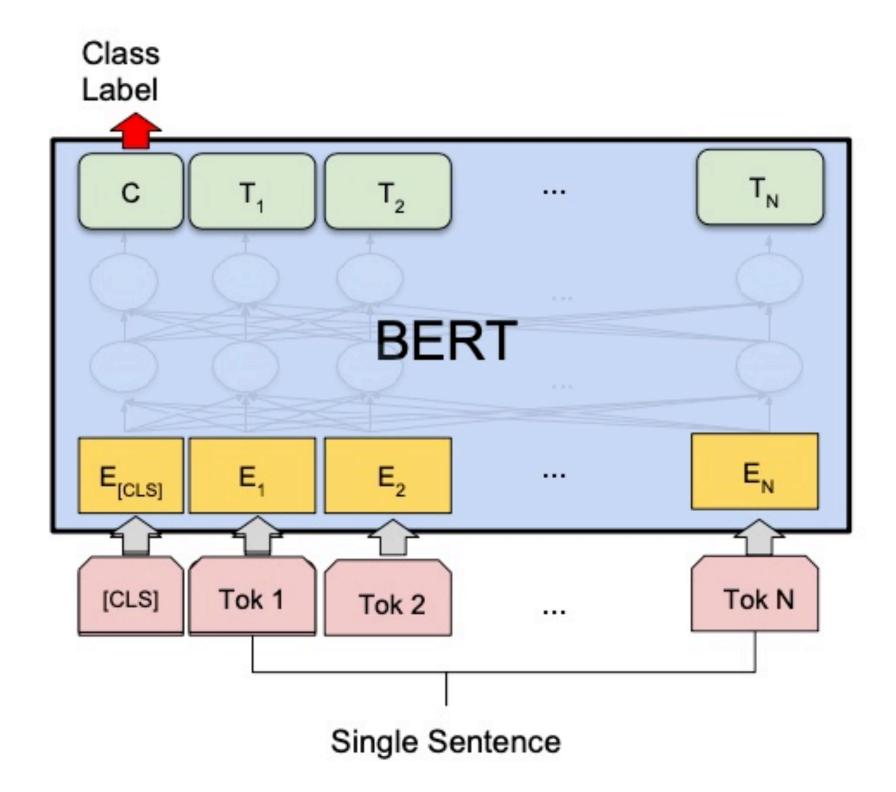


The pre-trained BERT encoder can be used directly for downstream tasks with **minimal task-specific parameters**!

BERT for text classification



- Add a classifier on top of the [CLS] representation
- New parameters: $d \times |C|$, jointly trained with BERT parameters
 - d = hidden dimension (e.g., 768)
 - |C| = number of classes (e.g., 2)



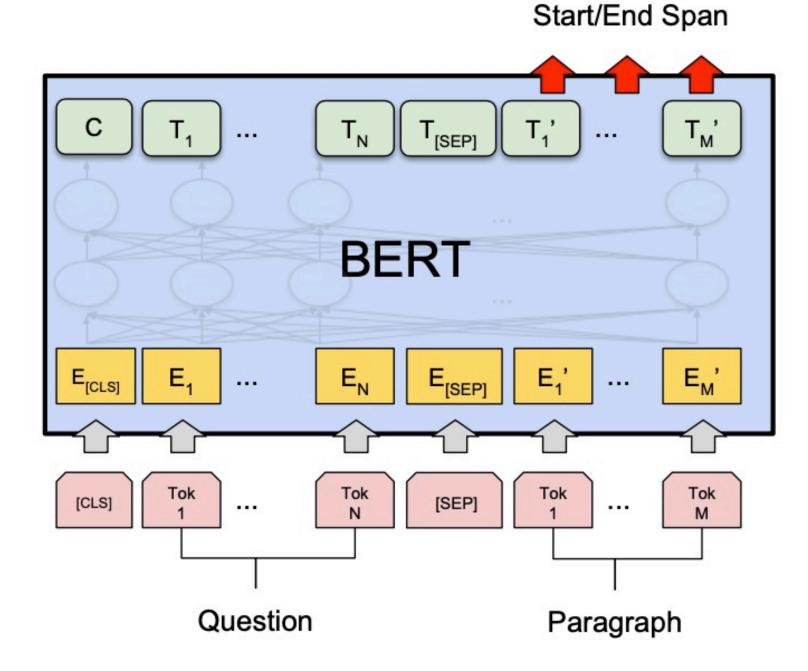
BERT for question answering

Stanford Question Answering Dataset (SQuAD)

Tesla was the fourth of five children. He had an older brother named Dane and three sisters, Milka, Angelina and Marica. Dane was killed in a horse-riding accident when Nikola was five. In 1861, Tesla attended the "Lower" or "Primary" School in Smiljan where he studied German, arithmetic, and religion. In 1862, the Tesla family moved to Gospić, Austrian Empire, where Tesla's father worked as a pastor. Nikola completed "Lower" or "Primary" School, followed by the "Lower Real Gymnasium" or "Normal School."

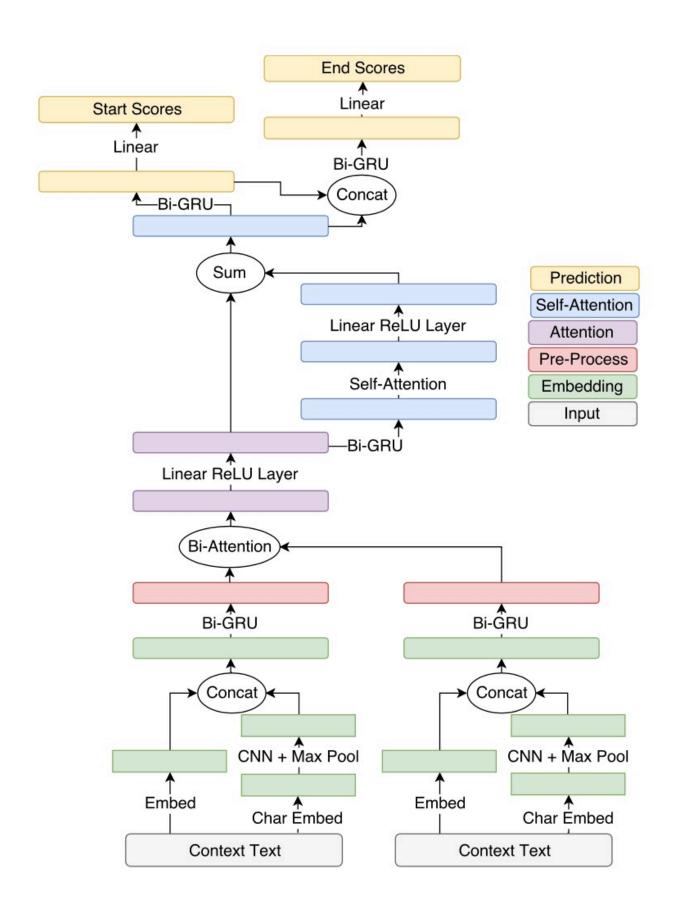
Q: What language did Tesla study while in school?

A: German

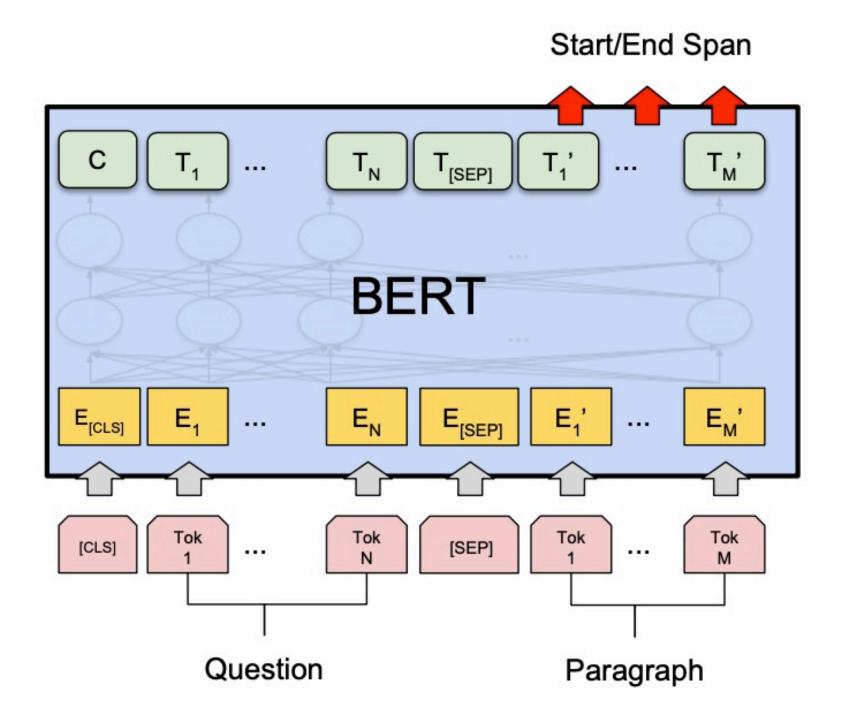


$$\mathcal{L} = -\log p_{\text{start}}(s^*) - \log p_{\text{end}}(e^*)$$
$$p_{\text{start}}(i) = \text{softmax}_i(\mathbf{w}_{\text{start}}^{\mathsf{T}} \mathbf{h}_i)$$
$$p_{\text{end}}(i) = \text{softmax}_i(\mathbf{w}_{\text{end}}^{\mathsf{T}} \mathbf{h}_i)$$

Pre-BERT QA models vs BERT



Only the word embeddings at the input layer are pre-trained (2-3M parameters in total)



All the parameters are pre-trained except for a small number of task-specific parameters $\mathbf{w}_{\text{start}}$, \mathbf{w}_{end}

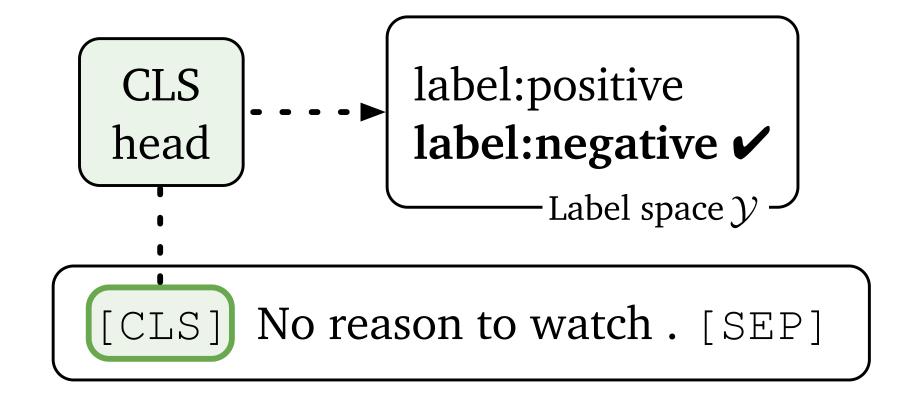
BERT has 110M or 330M parameters

Prompt-based fine-tuning

Task: sentiment classification

Input: "No reason to watch."

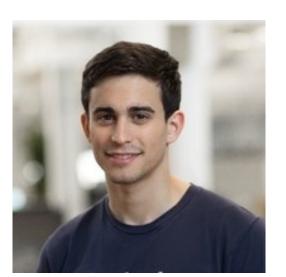
Output: positive 👍 or negative 👎 ?



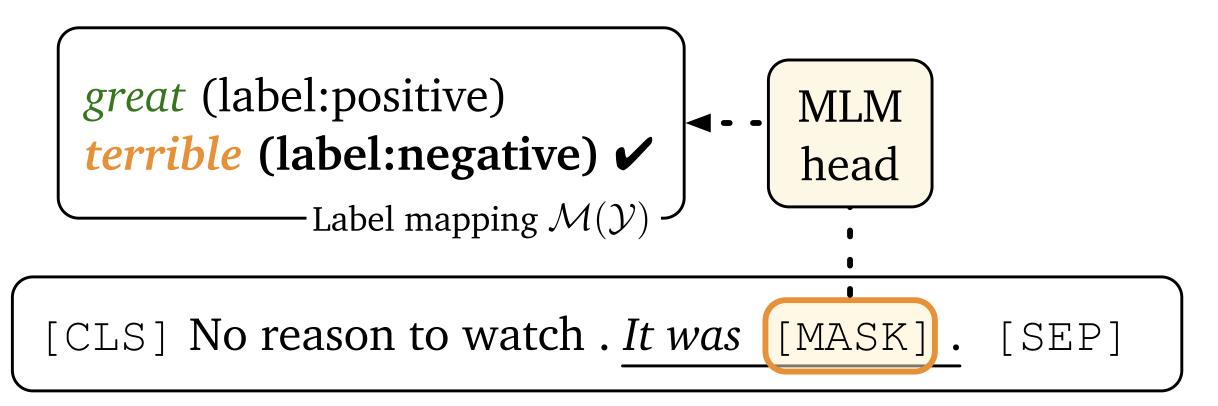
Fine-tuning

81.4% (32 examples)93.5% (67k examples; BERT-base)





Label mapping



Prompt-based fine-tuning

92.7% (32 examples)

BERT: training cost

- BERT-base: 12 layers, n = 512, d = 768, 110M parameters
- BERT-large: 24 layers, n = 512, d = 1024, 330M parameters
- Trained on Wikipedia + BooksCorpus (3.3 billion tokens)
- Estimate: 6 days for BERT-base and 26 days for BERT-large on 8 Nvidia Titan-V GPUs (12Gb memory)

by Jacob Portes*, Alex Trott*, Daniel King, Sam Havens
on March 9, 2023

MosaicBERT: Pretraining BERT from Scratch for \$20

https://www.mosaicml.com/blog/mosaicbert

(8x A100-80Gb GPUs)

RoBERTa

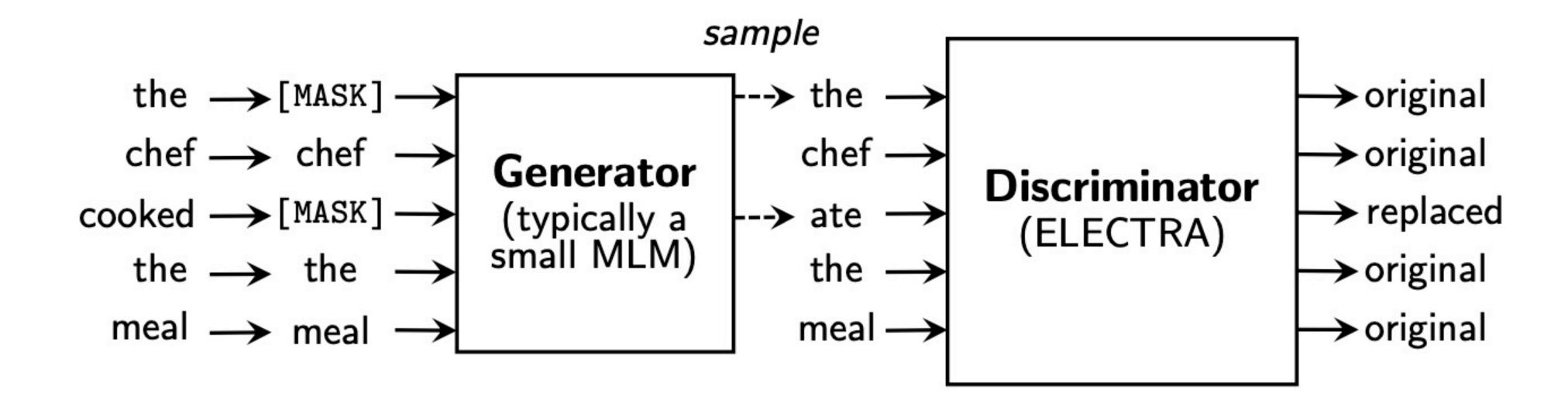
- BERT is still under-trained
- Removed the next-sentence prediction objective
- Trained longer with 10x data & bigger batch sizes
- Pre-trained on 1,024 V100 GPUs for one day in 2019

Model	data	bsz	steps	SQuAD (v1.1/2.0)	MNLI-m	SST-2
RoBERTa						
with BOOKS + WIKI	16GB	8K	100K	93.6/87.3	89.0	95.3
+ additional data (§3.2)	160GB	8K	100K	94.0/87.7	89.3	95.6
+ pretrain longer	160GB	8K	300K	94.4/88.7	90.0	96.1
+ pretrain even longer	160GB	8K	500K	94.6/89.4	90.2	96.4
$BERT_{LARGE}$						
with BOOKS + WIKI	13GB	256	1 M	90.9/81.8	86.6	93.7



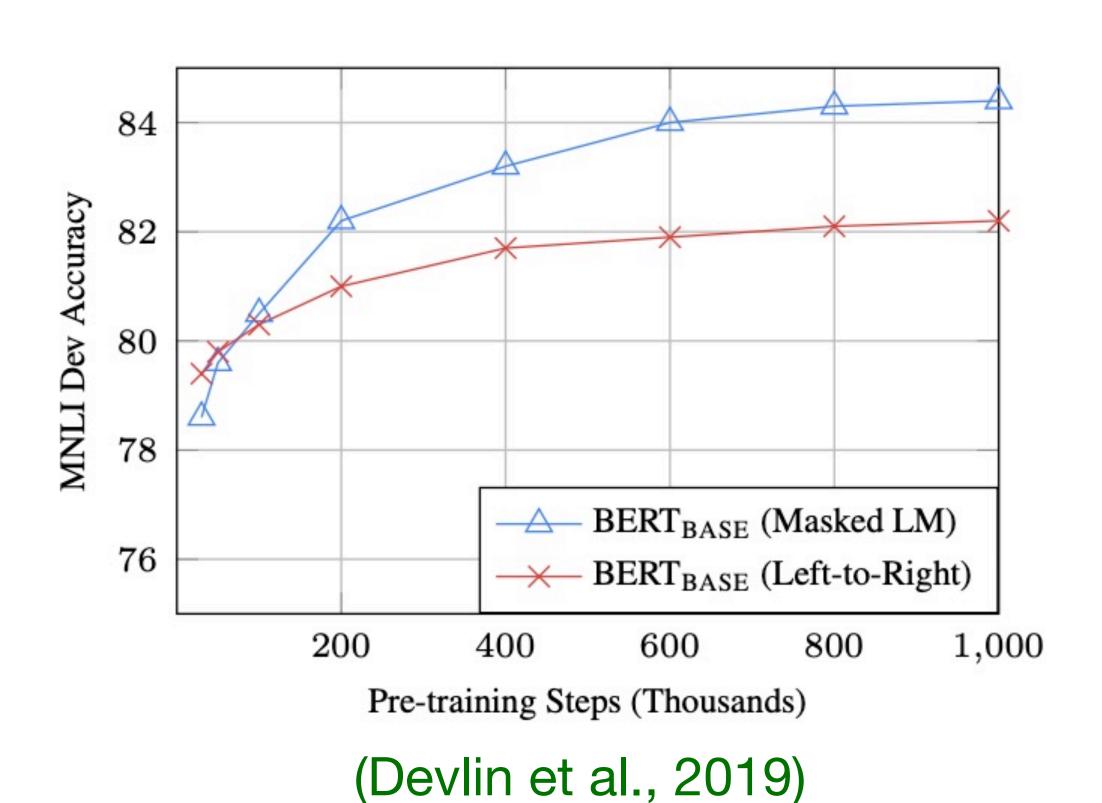
ELECTRA

ELECTRA provides a more **efficient** training method, because it predicts 100% of tokens (instead of 15%) every time



BERT vs GPT-1 models

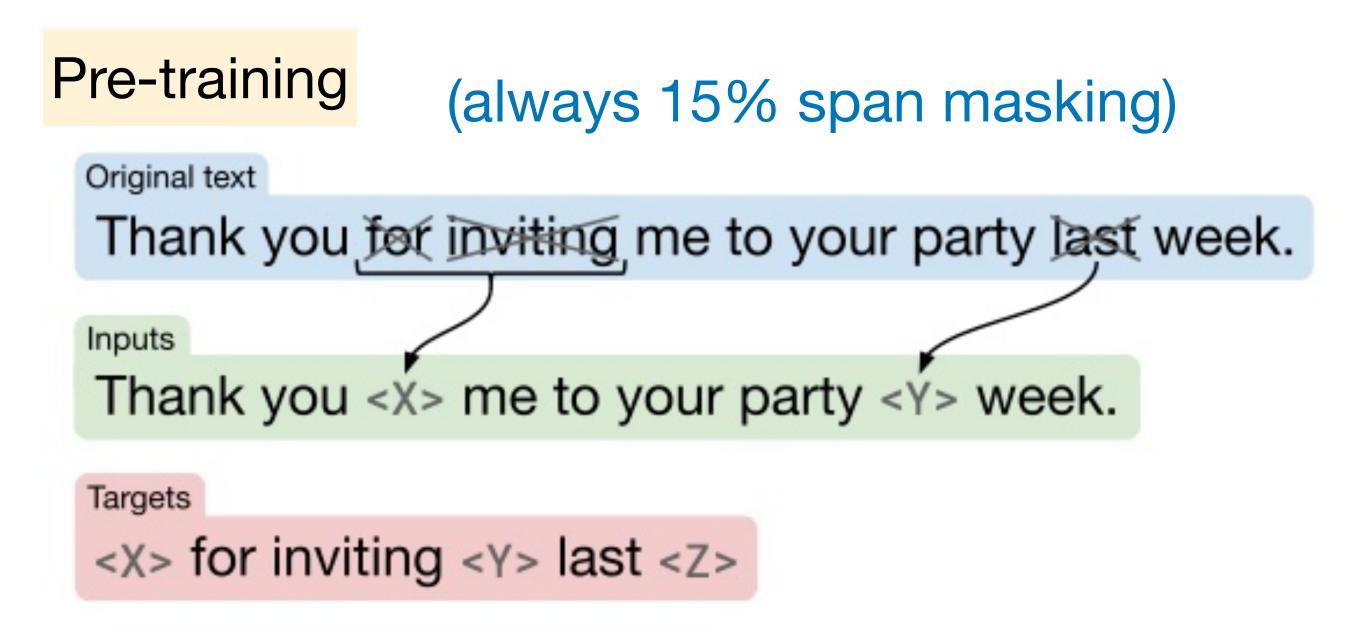
- Unlike GPT-1 (autoregressive language models), BERT/RoBERTa/ELECTRA can't generate text naturally!
- However, bidirectionality is important for natural language understanding tasks.



Why not combine the best of both worlds?

T5: Text-to-text models

- T5 = Text-to-Text Transfer Transformer
- Transformer encoder-decoder architecture
 - Encoder: preserves bidirectionality
 - Decoder: good for generation!

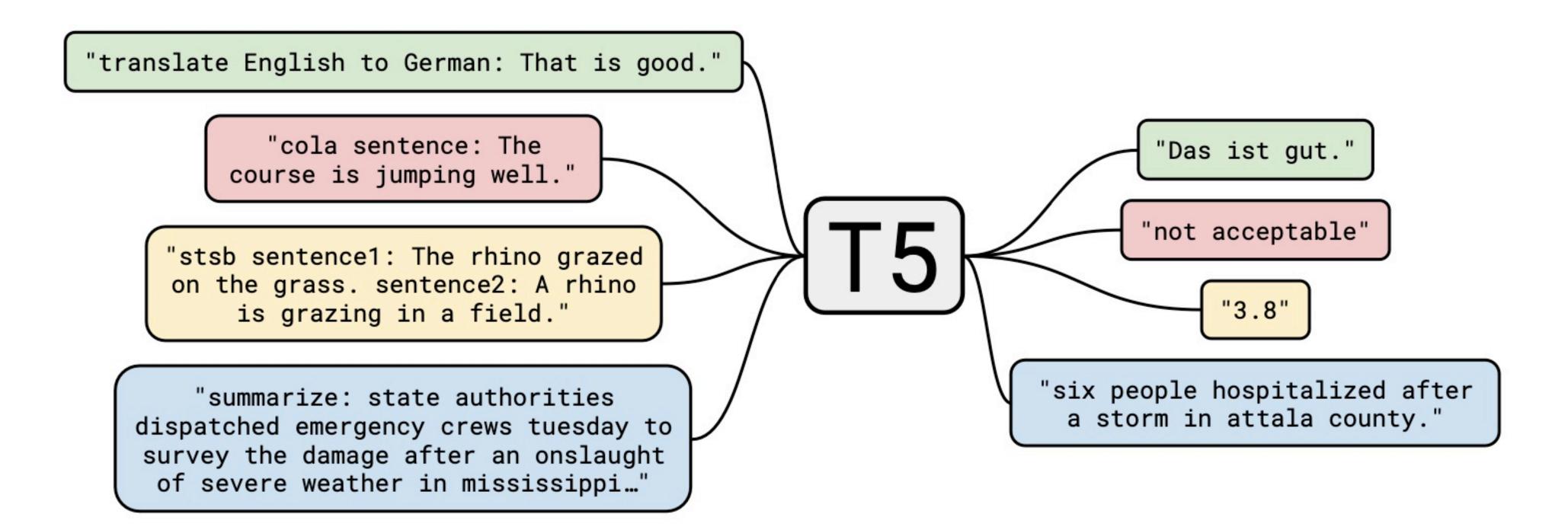


T5: Text-to-text models

• T5 = Text-to-Text Transfer Transformer

Fine-tuning

All NLU tasks can be cast as text prediction tasks too!



T5: Text-to-text models

• T5 = Text-to-Text Transfer Transformer

T5 comes in different sizes:

- t5-small.
- t5-base.
- t5-large.
- t5-3b.
- t5-11b.

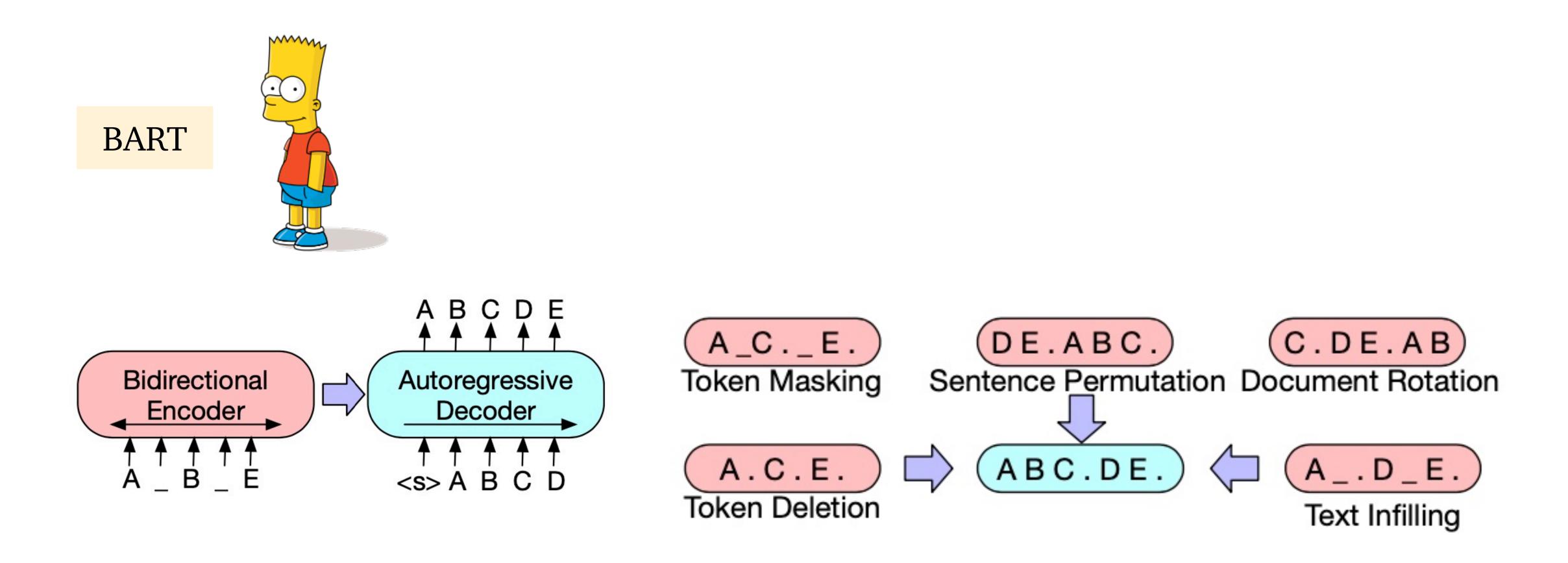
Training corpus:

C4 = Colossal Clean Crawled Corpus

A cleaned version of Common Crawl

Trained on 34B tokens

BART: another text-to-text model



GPT-3 The GPT-3 era: prompting and in-context learning

GPT-3

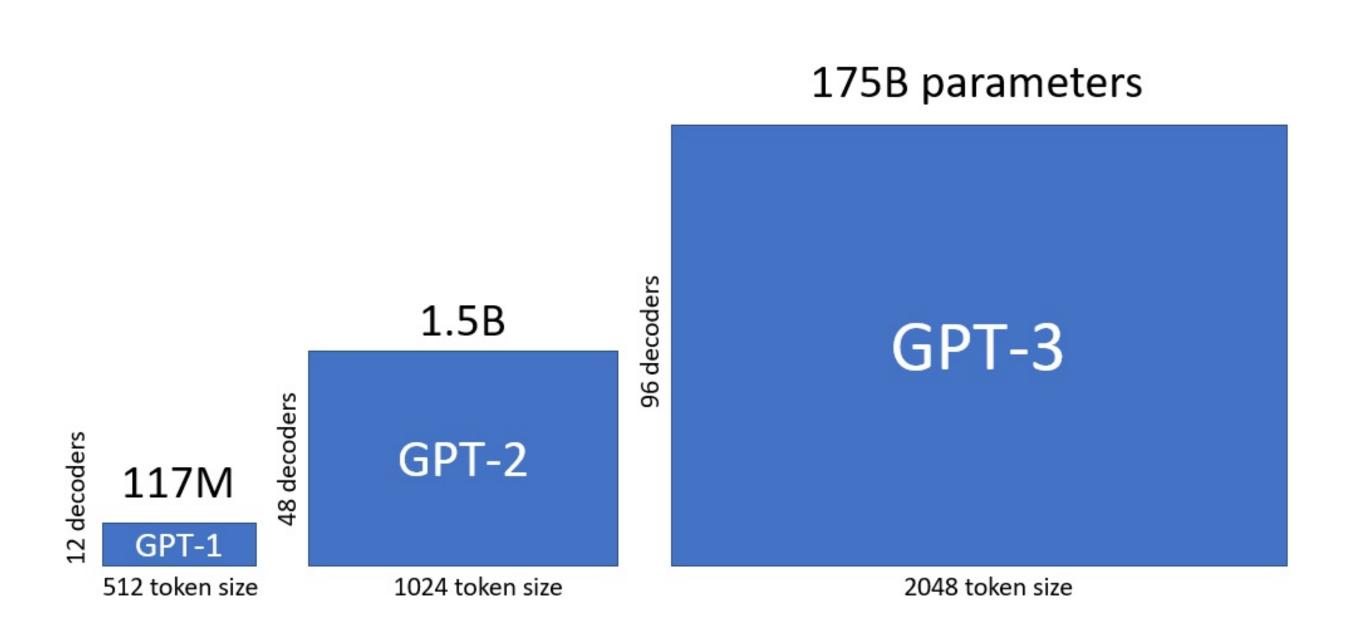
Language Models are Few-Shot Learners

Tom B. Brown* Benjamin Mann* Nick Ryder* Melanie Subbiah* Jared Kaplan[†] **Arvind Neelakantan Pranav Shyam Prafulla Dhariwal Girish Sastry** Sandhini Agarwal **Ariel Herbert-Voss Amanda Askell Gretchen Krueger Rewon Child Aditya Ramesh Tom Henighan** Daniel M. Ziegler Jeffrey Wu **Clemens Winter Christopher Hesse** Mark Chen **Eric Sigler Mateusz Litwin Scott Gray Benjamin Chess Jack Clark Christopher Berner** Dario Amodei Sam McCandlish **Alec Radford** Ilya Sutskever

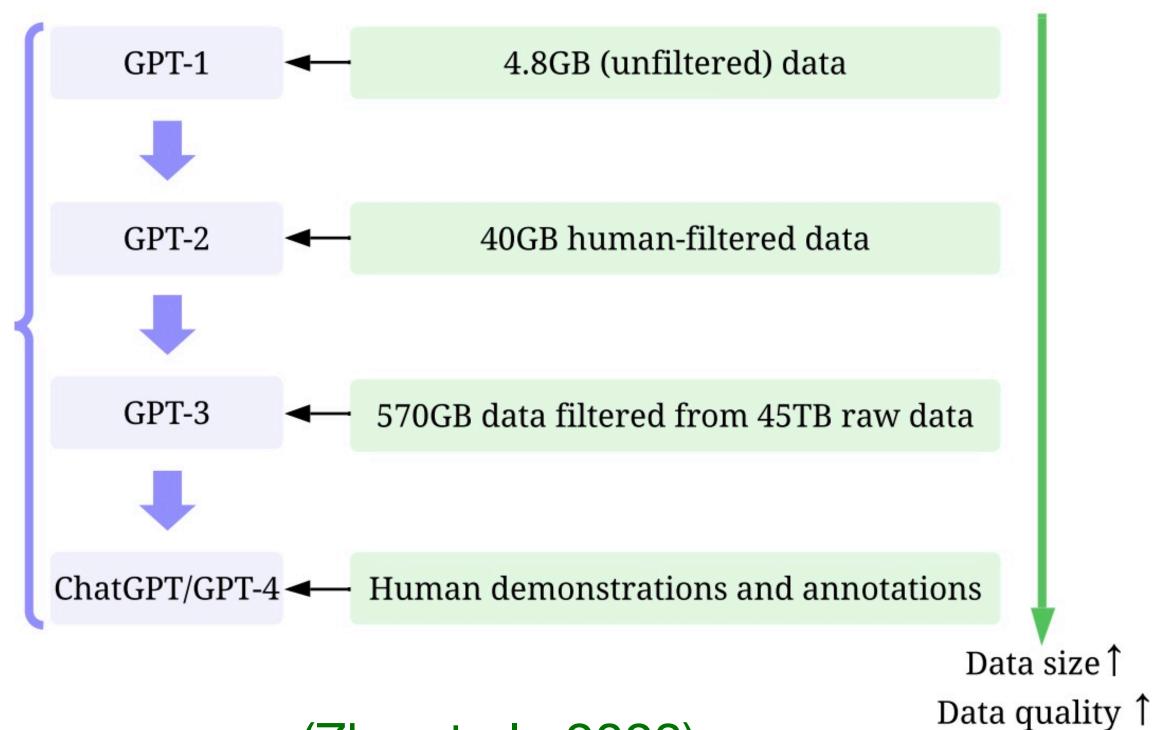
(Brown et al., 2020)

- From GPT-1 to GPT-2 to GPT-3
- Transformer decoder for autoregressive language modeling
- What's new?
 - Parameters ↑ Data ↑
 - These models can start generating human-like text!
 - You don't have to fine-tune these models - prompting works well!

GPT-1 vs GPT-2 vs GPT-3



https://medium.com/@YanAlx/



(Zha et al., 2023)

GPT-2 generation example

PROMPT (HUMAN-WRITTEN)

A train carriage containing controlled nuclear materials was stolen in Cincinnati today. Its whereabouts are unknown.

MODEL COMPLETION

The incident occurred on the downtown train line, which runs from Covington and Ashland stations.

In an email to Ohio news outlets, the U.S. Department of Energy said it is working with the Federal Railroad Administration to find the thief.

"The theft of this nuclear material will have significant negative consequences on public and environmental health, our workforce and the economy of our nation," said Tom Hicks, the U.S. Energy Secretary, in a statement. "Our top priority is to secure the theft and ensure it doesn't happen again."

The stolen material was taken from the University of Cincinnati's Research Triangle Park nuclear research site, according to a news release from Department officials.

GPT-3's in-context learning

- Before GPT-3, most use of pre-trained language models are through fine-tuning, on a reasonably-sized supervised dataset.
 - SST-2 has 67k examples, SQuAD has 88k (passage, answer, question) triples.
- GPT-3 shows that, with a very large autoregressive language model (175B parameters), the model can perform a task:
 - Using only a few examples "Few-shot learning"
 - Without gradient updates:
 the examples are only provided in the context! "In-context learning"

In-context learning

Few-shot

In addition to the task description, the model sees a few examples of the task. No gradient updates are performed.

```
Translate English to French:

sea otter => loutre de mer

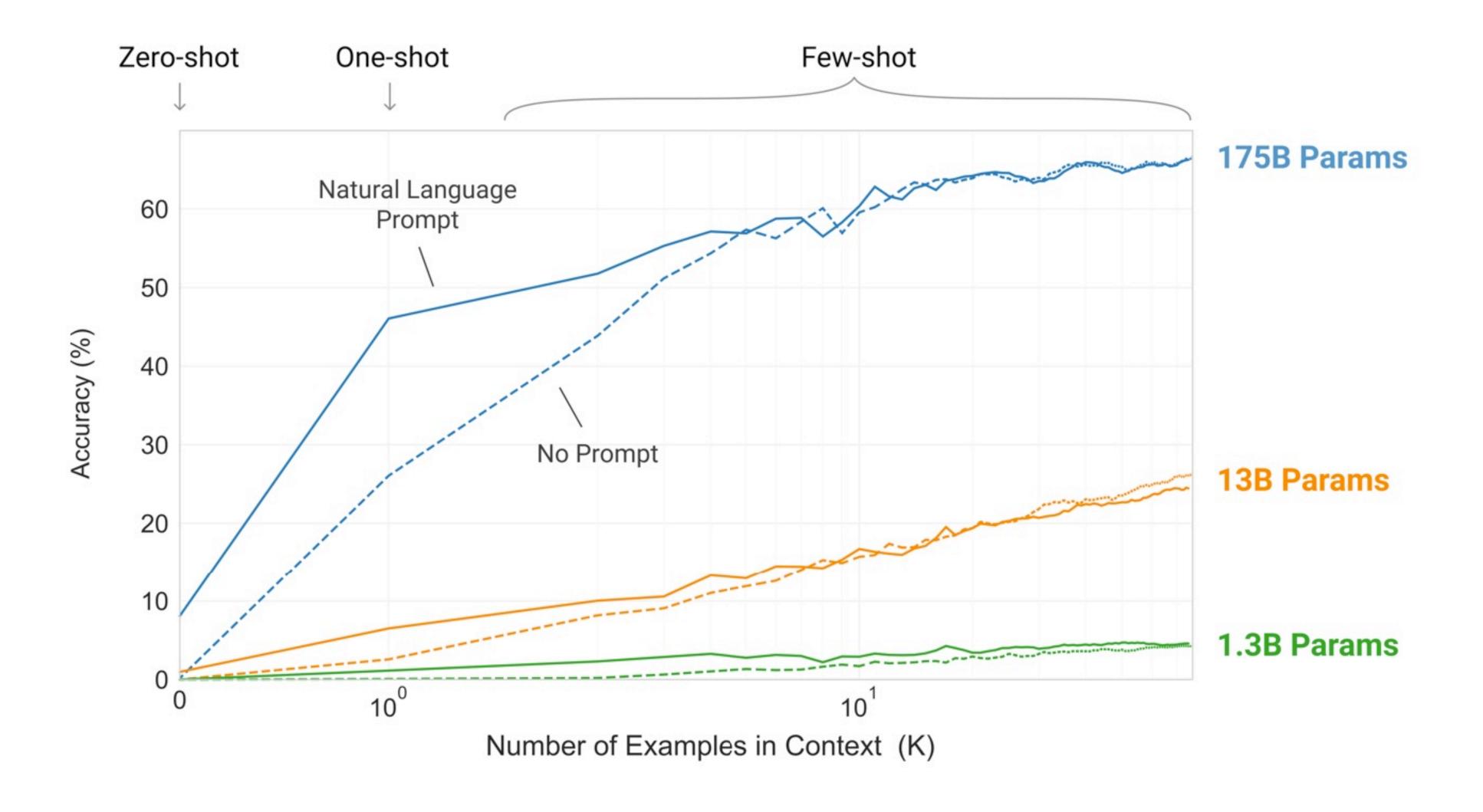
peppermint => menthe poivrée

plush girafe => girafe peluche

cheese => 

prompt
```

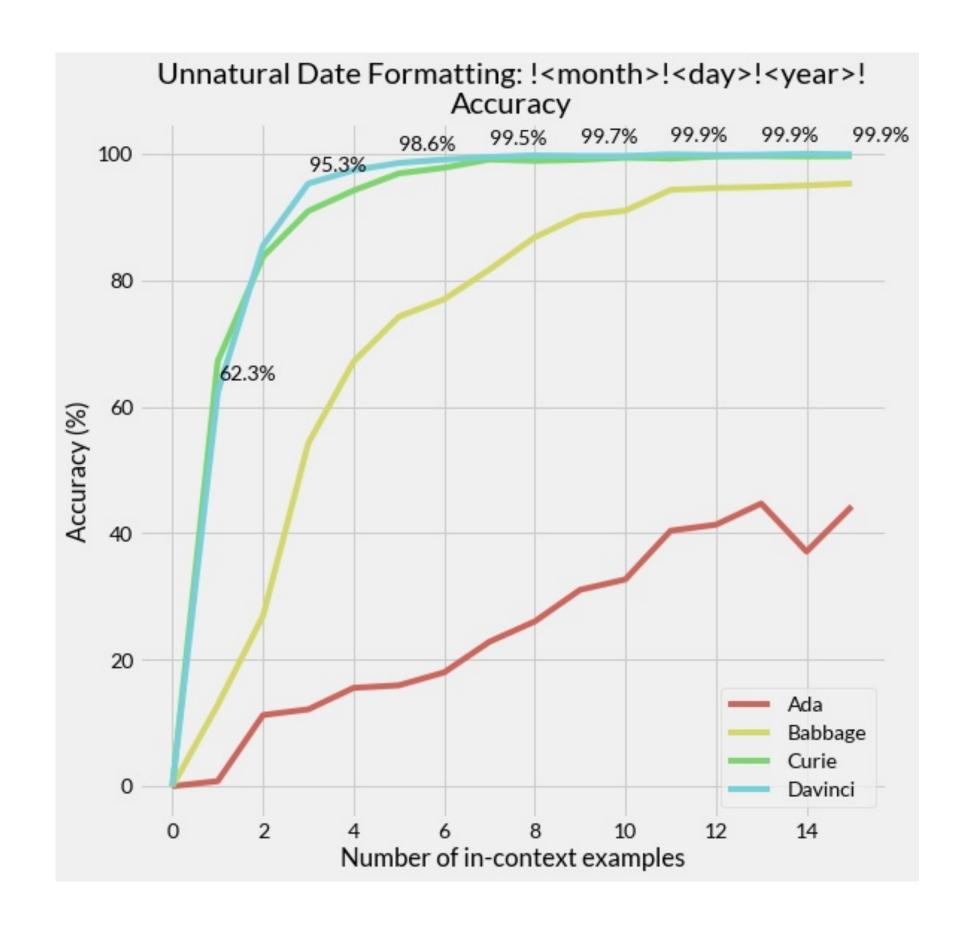
In-context learning



In-context learning

```
Input: 2014-06-01
Output: !06!01!2014!
Input: 2007-12-13
Output: !12!13!2007!
Input: 2010-09-23
Output: !09!23!2010!
Input: 2005-07-23
Output: !07!23!2005!

L - - model completion
```



Understanding in-context learning

• Hypothesis #1: Transformers perform implicit gradient descent to update an "inner model"

Transformers Learn In-Context by Gradient Descent

Johannes von Oswald ¹² Eyvind Niklasson ² Ettore Randazzo ² João Sacramento ¹ Alexander Mordvintsev ² Andrey Zhmoginov ² Max Vladymyrov ²

Why Can GPT Learn In-Context?
Language Models Implicitly Perform Gradient Descent as
Meta-Optimizers

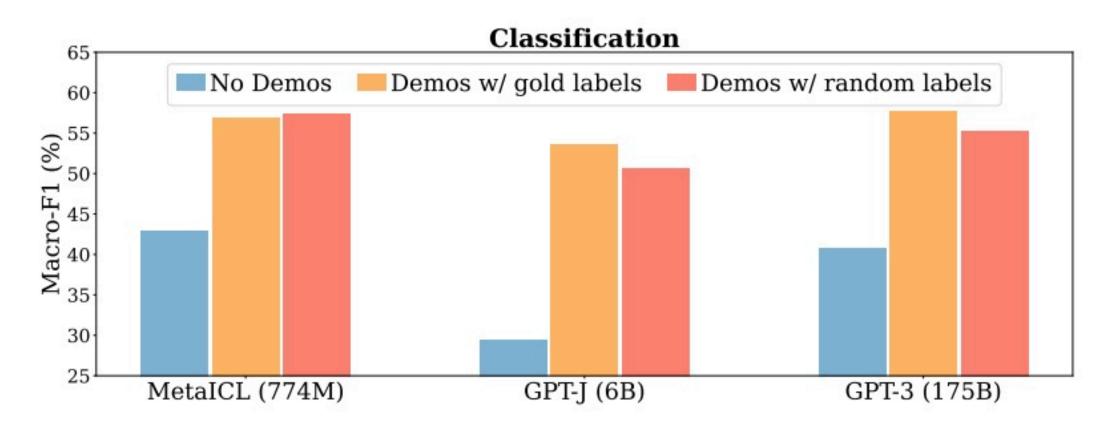
Damai Dai[†]*, Yutao Sun^{||}*, Li Dong[‡], Yaru Hao[‡], Shuming Ma[‡], Zhifang Sui[†], Furu Wei[‡]

 Hypothesis #2: Transformers learn tasks required for downstream applications during pretraining, and in-context demonstrations are only used to recognize which task is required

Rethinking the Role of Demonstrations: What Makes In-Context Learning Work?

Sewon Min^{1,2} Xinxi Lyu¹ Ari Holtzman¹ Mikel Artetxe² Mike Lewis² Hannaneh Hajishirzi^{1,3} Luke Zettlemoyer^{1,2}

¹University of Washington ²Meta AI ³Allen Institute for AI {sewon, alrope, ahai, hannaneh, lsz}@cs.washington.ed {artetxe, mikelewis}@meta.com



Ground-truth labels don't matter!

Understanding in-context learning

We disentangle In-context learning into two roles - task recognition (TR) vs task learning (TL)

- TR: recognizes the task from demonstrations and applies LLMs' pre-trained priors
- TL: learns a new input-label mapping from demonstrations
- ICL performs both TR and TL, but TL emerges with larger models and more demonstrations



Chain-of-thought (CoT) prompting

Standard Prompting

Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

Chain of Thought Prompting

Input

Q: Roger has 5 tennis balls. He buys 2 more cans of tennis balls. Each can has 3 tennis balls. How many tennis balls does he have now?

A: Roger started with 5 balls. 2 cans of 3 tennis balls each is 6 tennis balls. 5 + 6 = 11. The answer is 11.

Q: The cafeteria had 23 apples. If they used 20 to make lunch and bought 6 more, how many apples do they have?

Model Output

A: The answer is 27.



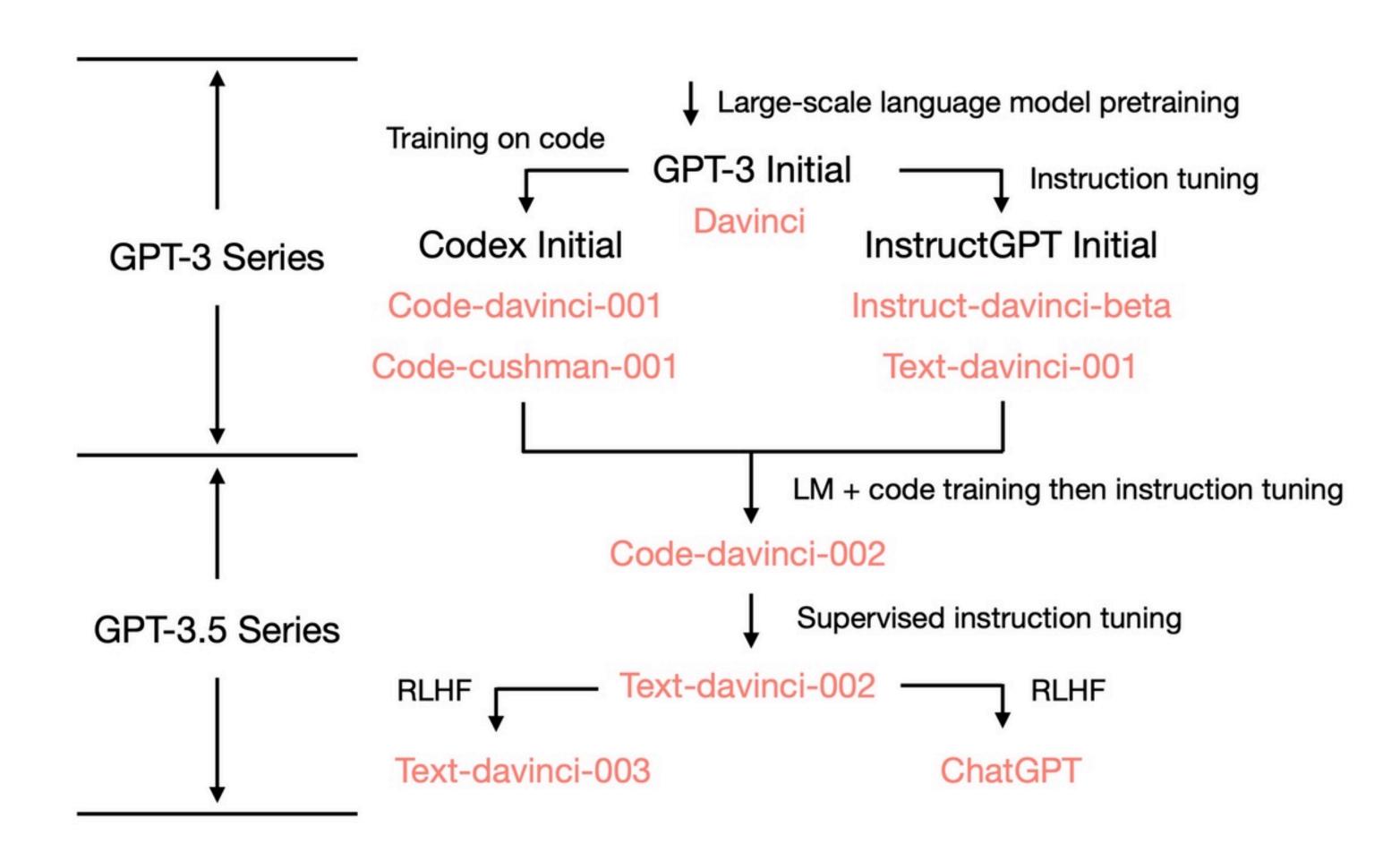
Model Output

A: The cafeteria had 23 apples originally. They used 20 to make lunch. So they had 23 - 20 = 3. They bought 6 more apples, so they have 3 + 6 = 9. The answer is 9. 🗸



ChatGPT The ChatGPT era: Supervised instruction tuning and RLHF

From 2020's GPT-3 to 2022's ChatGPT



What's new?

- Code training
- Supervised instruction tuning
- RLHF = Reinforcement learning from human feedback

https://yaofu.notion.site/How-does-GPT-Obtain-its-Ability-Tracing-Emergent-Abilities-of-Language-Models-to-their-Sources-b9a57ac0fcf74f30a1ab9e3e36fa1dc1

Post-training pipeline of LLMs

Training language models to follow instructions with human feedback

Long Ouyang* Jeff Wu* Xu Jiang* Diogo Almeida* Carroll L. Wainwright*

Pamela Mishkin* Chong Zhang Sandhini Agarwal Katarina Slama Alex Ray

John Schulman Jacob Hilton Fraser Kelton Luke Miller Maddie Simens

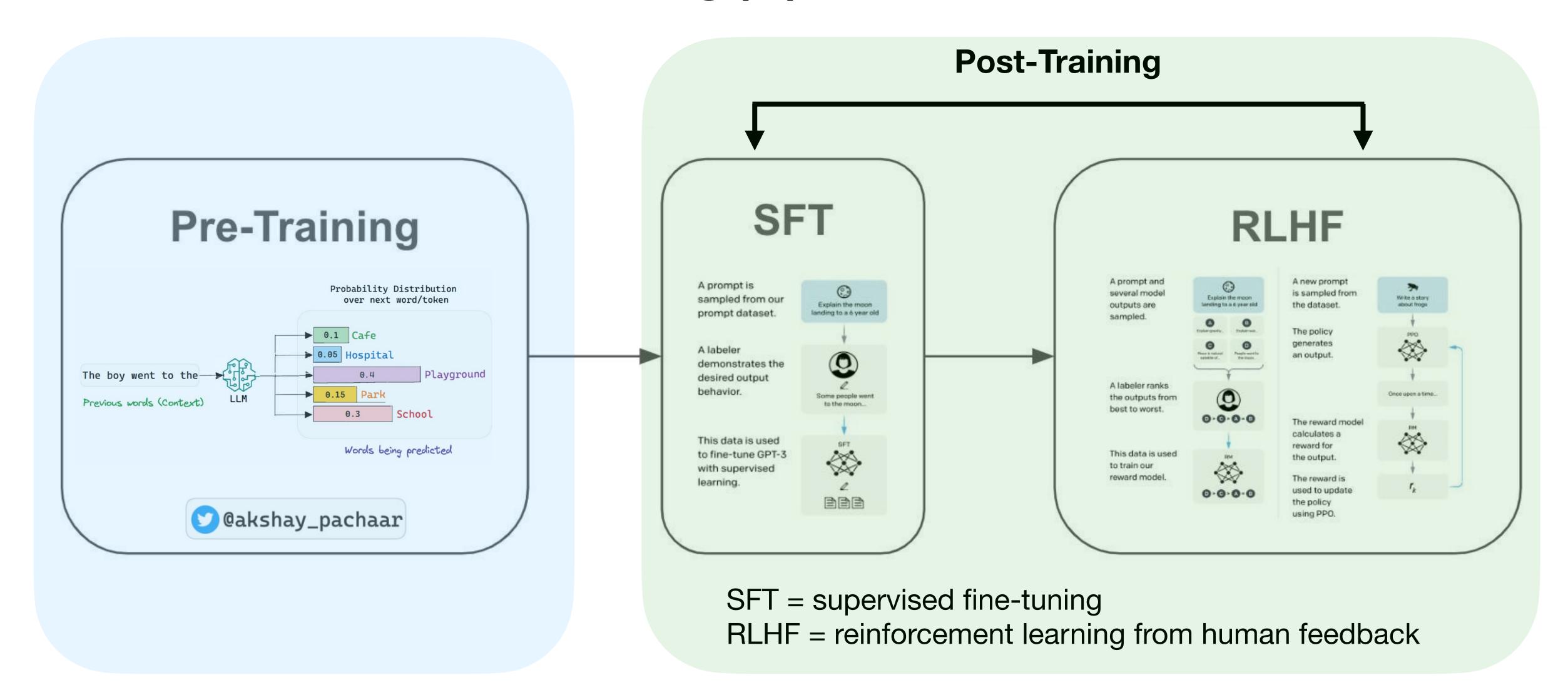
Amanda Askell[†] Peter Welinder Paul Christiano*[†]

Jan Leike* Ryan Lowe*

OpenAI

The InstructGPT paper

Post-training pipeline of LLMs



Supervised fine-tuning (SFT)

- Instruction data: (instruction, response)
- Objective: next-token prediction conditioned on instruction

Can you write a short introduction about the relevance of the term "monopsony" in economics? Please use examples related to potential monopsonies in the labour market and cite relevant research.

"Monopsony" refers to a market structure where there is only one buyer for a particular good or service. In economics, this term is particularly relevant in the labor market, where a monopsony employer has significant power over the wages and working conditions of their employees. The presence of a monopsony can result in lower wages and reduced employment opportunities for workers, as the employer has little incentive to increase wages or provide better working conditions understand the extent and impact of monopsonies on the economy and to develop policies to address this issue. References: Bivens, J., & Mishel, L. (2013). The Pay of Corporate Executives and Financial Professionals as Evidence of Rents in Top 1 Percent Incomes. Journal of Economic Perspectives, 27(3), 57-78.

Supervised fine-tuning (SFT)

Use-case	Prompt	Use-case	(%)
Brainstorming	List five ideas for how to regain enthusiasm for my	Generation	45.6%
	career	Open QA	12.4%
Generation	Write a short story where a beer goes to the beach	Brainstorming	11.2%
Generation		Chat	8.4%
	makes friends with a seal, and then returns home.	Rewrite	6.6%
Rewrite	This is the summary of a Broadway play:	Summarization	4.2%
	"""	Classification	3.5%
"""	{summary}	Other	3.5%
		Closed QA	2.6%
	This is the outline of the commercial for that play:	Extract	1.9%

SFT data: only ~13k (written by labeller, not public)

Collection of supervised fine-tuning data

- Repurposed from existing supervised datasets
- Human-written instructions and responses



Super-NaturalInstructions (Wang et al., 2022)

Also: FLAN, T0++

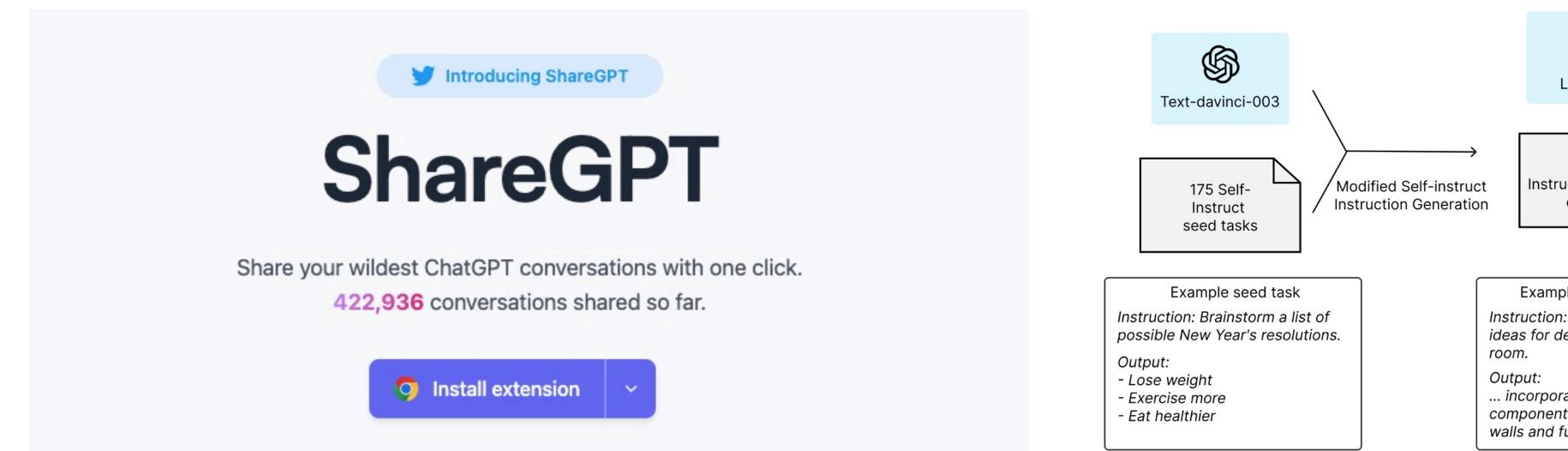




OpenAssisstant (Kopf et al., 2023)

Collection of supervised fine-tuning data

- Response distilled from GPT models
- Instructions can be generated by GPT models too, e.g., Self-Instruct (Wang et al., 2023)

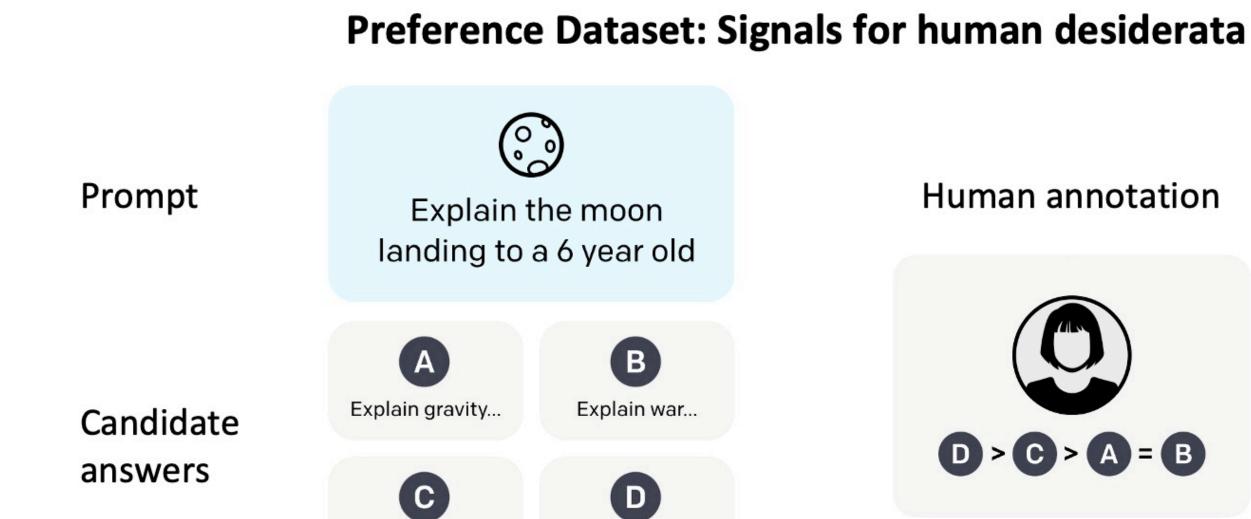


https://sharegpt.com/

Stanford Alpaca

Learning from human feedback

• Preference data: (instruction, winning response, losing response)



People went to

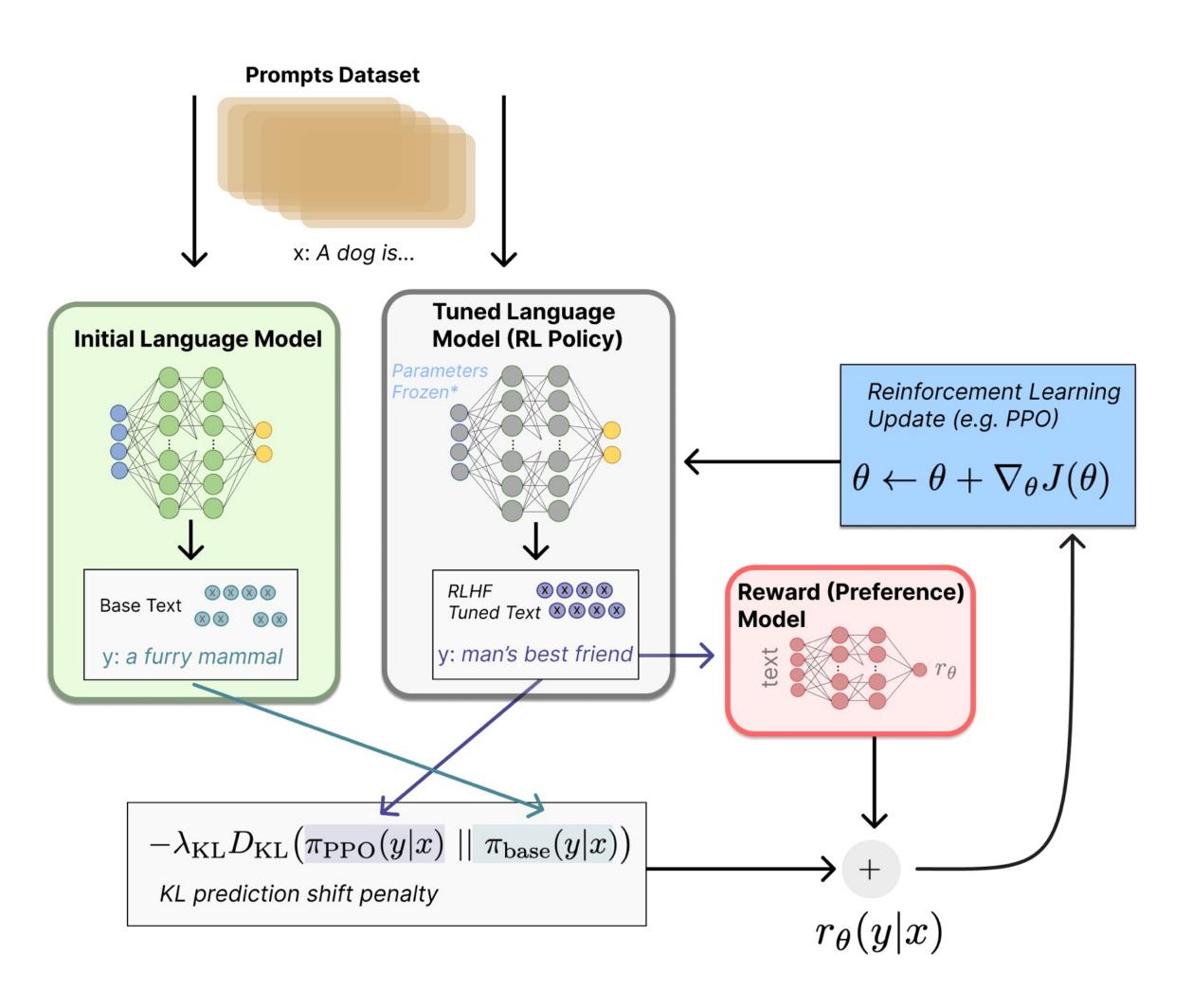
the moon...

Moon is natural

satellite of...

InstructGPT: 33k prompts, each with K (4~9) corresponding SFT model completions ranked by labellers

Reinforcement learning from human feedback

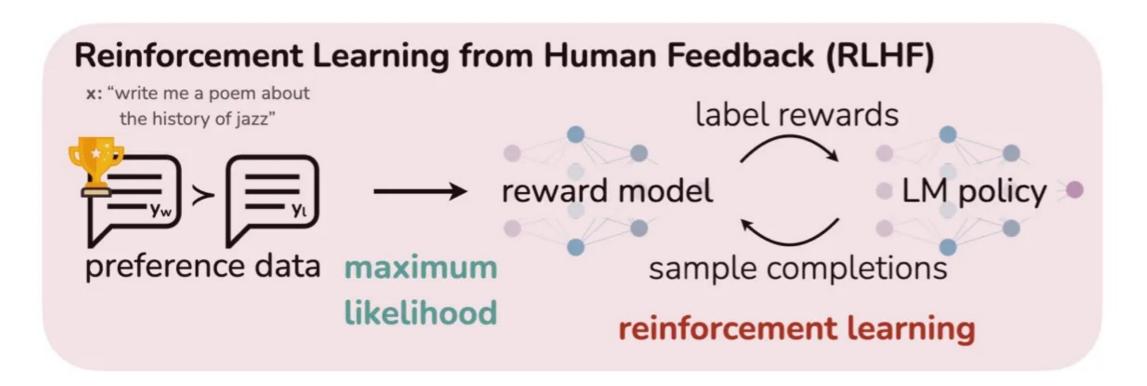


Multiple stages in training

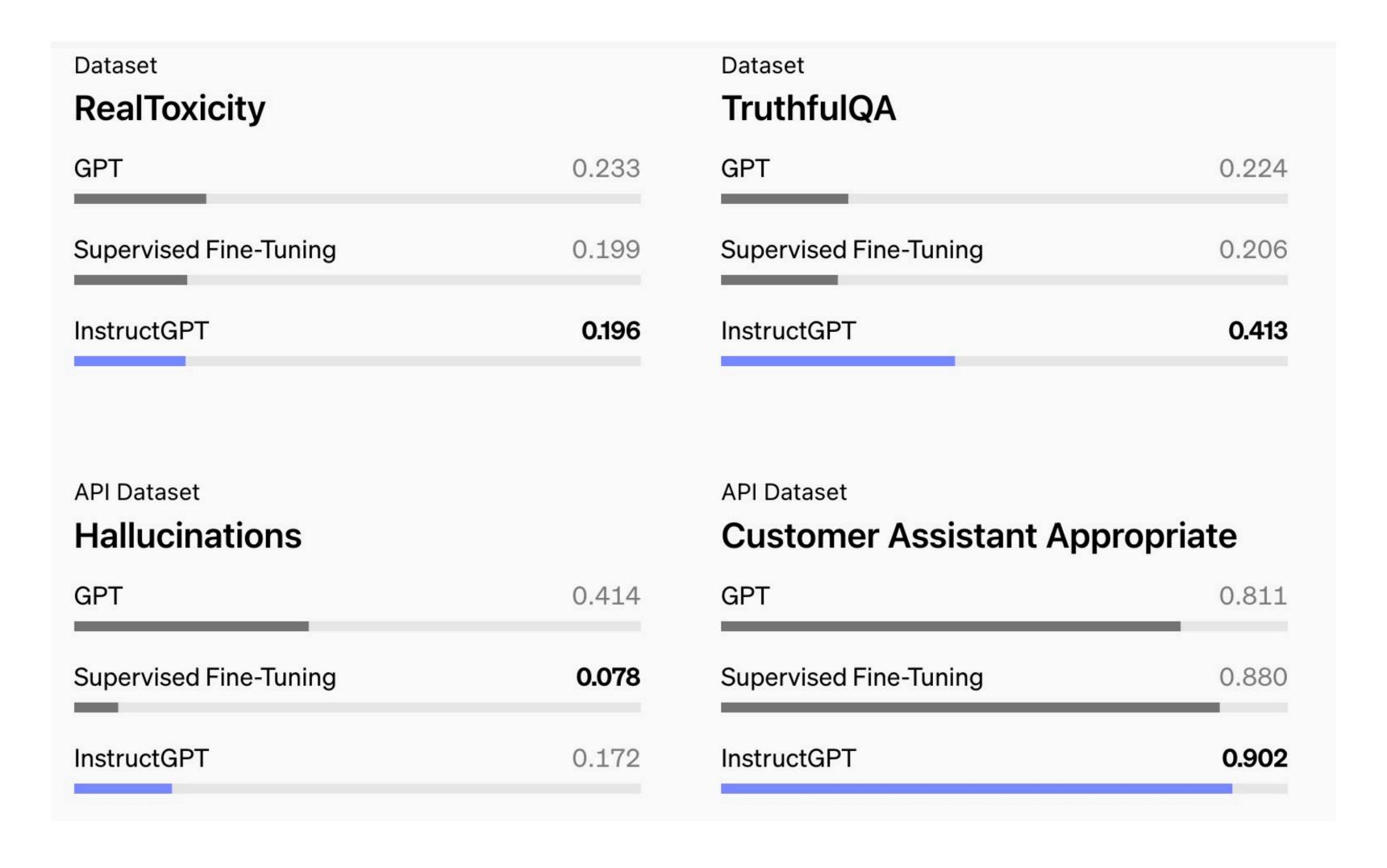
- Reward model training
- Sampling from policy model
- Policy model update

Multiple models involved

- Reward model
- Policy model
- Reference model

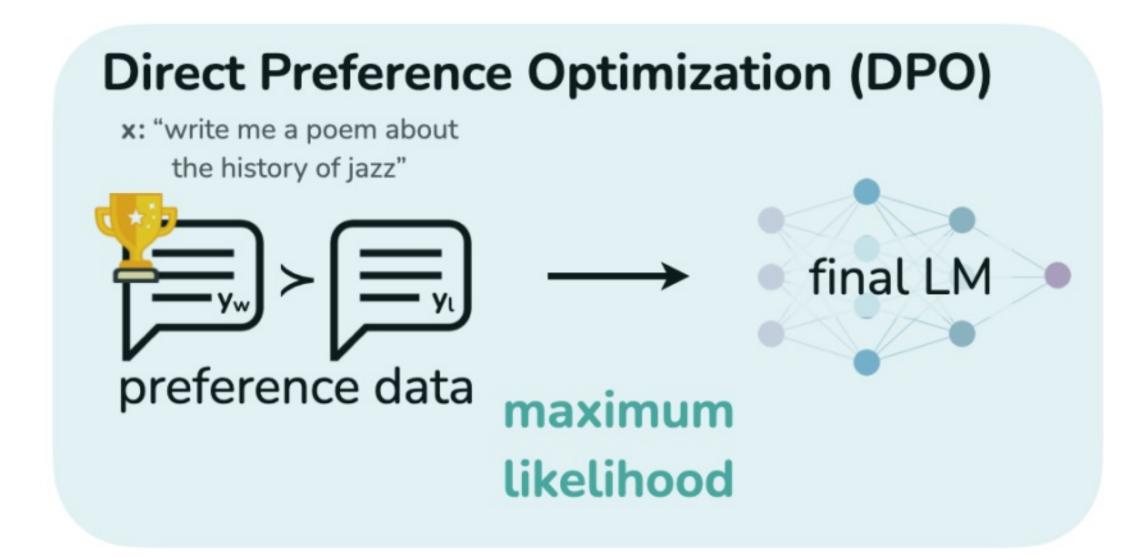


GPT-3 vs InstructGPT



Direct preference optimization (DPO)

Instead of training an explicit reward model, express reward in the form of policy model:



Implicit reward expression:

$$r(x, y) = \beta \log \frac{\pi_{\theta}(y \mid x)}{\pi_{\text{ref}}(y \mid x)} + \beta \log Z(x)$$

Bradley-Terry ranking objective:

$$\mathcal{L}_R(r_{\phi}, \mathcal{D}) = -\mathbb{E}_{(x, y_w, y_l) \sim \mathcal{D}} \left[\log \sigma(r_{\phi}(x, y_w) - r_{\phi}(x, y_l)) \right]$$

DPO objective:

$$\mathcal{L}_{\text{DPO}}(\pi_{\theta}; \pi_{\text{ref}}) = -\mathbb{E}_{(x, y_w, y_l) \sim \mathcal{D}} \left[\log \sigma \left(\beta \log \frac{\pi_{\theta}(y_w \mid x)}{\pi_{\text{ref}}(y_w \mid x)} - \beta \log \frac{\pi_{\theta}(y_l \mid x)}{\pi_{\text{ref}}(y_l \mid x)} \right) \right]$$

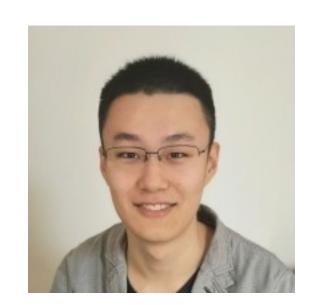
Simple preference optimization (SimPO)

$$egin{aligned} \mathcal{L}_{\mathbf{DPO}}(\pi_{ heta};\pi_{\mathrm{ref}}) = \ -\mathbb{E}igg[\log\sigmaigg(eta\lograc{\pi_{ heta}(y_w\mid x)}{\pi_{\mathrm{ref}}(y_w\mid x)} - eta\lograc{\pi_{ heta}(y_l\mid x)}{\pi_{\mathrm{ref}}(y_l\mid x)}igg)igg] \end{aligned}$$

$$egin{aligned} \mathcal{L}_{\mathbf{SimPO}}(\pi_{ heta}) = \ -\mathbb{E}igg[\log\sigmaigg(rac{eta}{|y_w|}\log\pi_{ heta}(y_w\mid x) - rac{eta}{|y_l|}\log\pi_{ heta}(y_l\mid x) - \gammaigg)igg] \end{aligned}$$

• A simple length-normalized reward (reference-free!):

$$r_{\text{SimPO}}(x, y) = \frac{\beta}{|y|} \log \pi_{\theta}(y \mid x) = \frac{\beta}{|y|} \sum_{i=1}^{|y|} \log \pi_{\theta}(y_i \mid x, y_{< i})$$





Introducing target reward margin in Bradley-Terry objective:

$$p(y_w \succ y_l \mid x) = \sigma\left(r(x, y_w) - r(x, y_l) - \gamma\right)$$

Simple preference optimization (SimPO)

