Learning Structured Predictors

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DMETRICS

https://dmetrics.com

Outline

Supervised (Structured) Prediction

- Learning to predict: given training data

$$\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\}$$

  learn a predictor $\mathbf{x} \to \mathbf{y}$ that *works well* on unseen inputs $\mathbf{x}$

- Non-Structured Prediction: outputs $\mathbf{y}$ are atomic
  - Binary classification: $\mathbf{y} \in \{-1, +1\}$
  - Multiclass classification: $\mathbf{y} \in \{1, 2, \ldots, L\}$

- Structured Prediction: outputs $\mathbf{y}$ are structured
  - Sequence prediction: $\mathbf{y}$ are sequences
  - Parsing: $\mathbf{y}$ are trees
  - ...

Named Entity Recognition

$\mathbf{y}$	PER	-	QNT	-	-	ORG	ORG	-	TIME
$\mathbf{x}$	Jim	bought	300	shares	of	Acme	Corp.	in	2006

Named Entity Recognition

y	PER	-	QNT	-	-	ORG	ORG	-	TIME
x	Jim	bought	300	shares	of	Acme	Corp.	in	2006

y	PER	PER	-	-	LOC
x	Jack	London	went	to	Paris

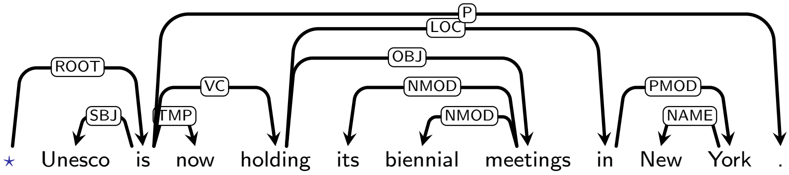
y	PER	PER	-	-	LOC
x	Paris	Jackson	went	to	London

y	PER	-	-	LOC
x	Jackie	went	to	Lisdon

Part-of-speech Tagging

$\mathbf{y}$	NOUN	NOUN	VERB	NOUN	.
$\mathbf{x}$	Fruit	flies	like	bananas	.

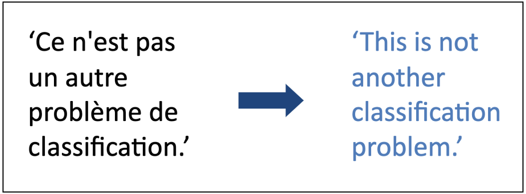
Syntactic Dependency Parsing



$\mathbf{x}$ are sentences

$\mathbf{y}$ are syntactic dependency trees

Machine Translation



(illustration by Ben Taskar)

$\mathbf{x}$ are sentences in some source language (e.g. French)

$\mathbf{y}$ are sentence translations in a target language (e.g. English)

Object Detection

(Kumar and Hebert, 2003)

$\mathbf{x}$ are images

$\mathbf{y}$ are grids labeled with object types

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Today's Goals

- Introduce basic concepts for structured prediction
  - We will focus on sequence prediction

- What can we can borrow from standard classification?
  - Learning paradigms and algorithms, in essence, work here too
  - However, computations behind algorithms are prohibitive

- Today's main topics:
  - Transition systems versus factored models
  - Feature representations of structured input-output pairs
  - Prediction algorithms
  - Learning algorithms: Perceptron and CRF
  - Local and global learning losses
- Topics not covered:
  - NLP task overviews, evaluation, state-of-the-art systems
  - Hidden (structured) representations
  - Unsupervised learning (induction of labeled sequences and trees)

Outline

Sequence Prediction

$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

Sequence Prediction

- $\mathbf{x} = x_1 x_2 \ldots x_n$ are input sequences, $x_i \in \mathcal{X}$
- $\mathbf{y} = y_1 y_2 \ldots y_n$ are output sequences, $y_i \in \{1, \ldots, L\}$

- **Goal:** given training data

$$\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\}$$

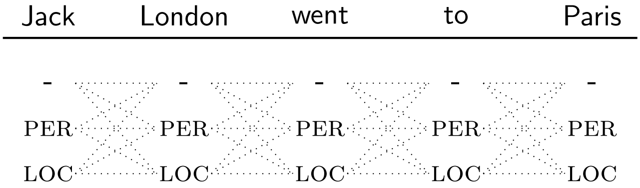
  learn a predictor $\mathbf{x} \rightarrow \mathbf{y}$ that works well on unseen inputs $\mathbf{x}$

- What is the form of our prediction model?

Exponentially-many Solutions

- Let $\mathcal{Y} = \{\text{-}, \textsc{per}, \textsc{loc}\}$

- The solution space (all output sequences):



  - Each path is a possible solution

- For an input sequence of size n, there are $|\mathcal{Y}|^n$ possible outputs

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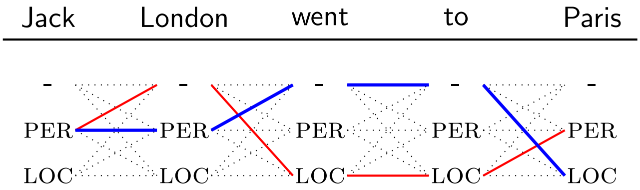

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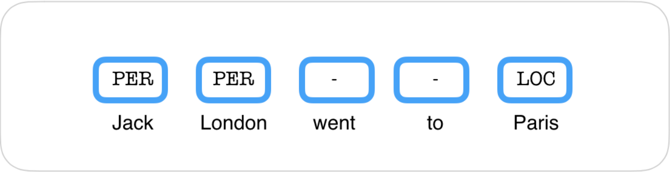
Approach 1: Label Classifiers



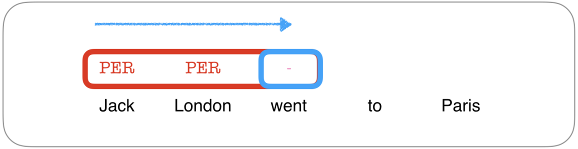
- Scoring of individual labels at each position

$$\hat{y}_t = \underset{l \,\in\, \{\text{LOC},\ \text{PER},\ \text{-}\}}{\operatorname{argmax}} \ \operatorname{score}(\mathbf{x}, t, l)$$

- For linear models, $\operatorname{score}(\mathbf{x}, t, l) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, t, l)$
  - $\mathbf{f}(\mathbf{x}, t, l) \in \mathbb{R}^d$ represents an assignment of label l for x_t
  - $\mathbf{w} \in \mathbb{R}^d$ is a vector of parameters (learned), has a weight for each feature in $\mathbf{f}$
- Can capture interactions between full input $\mathbf{x}$ and one output label l

  e.g.: current word, surrounding words, capitalization, prefix-suffix, gazetteer, ...
- Can not capture interactions between output labels!

Approach 2: Transition-based Sequence Prediction

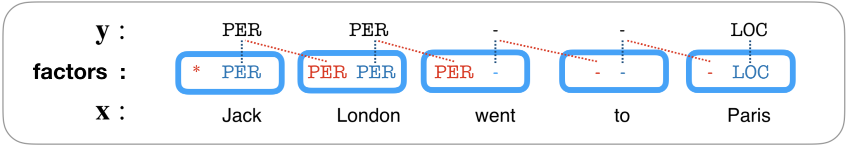


- Score one label at a time, left-to-right, conditioning on previous predictions:

$$\hat{y}_t = \underset{l \in \{\text{LOC}, \text{PER}, \text{-}\}}{\text{argmax}} \; \text{score}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1})$$

- Captures interactions between full input $\mathbf{x}$ and prefixes of the output sequence
- Greedy predictions, prone to search errors even with beam search
- Why left-to-right and not right-to-left?

Approach 3: Factored Sequence Prediction



- Scoring of label bigrams (pairs of adjacent labels) at each position:

$$\hat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{Y}^n}{\operatorname{argmax}} \operatorname{score}(\mathbf{x}, \mathbf{y}) = \underset{\mathbf{y} \in \mathcal{Y}^n}{\operatorname{argmax}} \sum_{i=1}^{n} \operatorname{score}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Output sequence factored into label bigrams
- Captures interactions between full input $\mathbf{x}$ and factors of output sequence
- Prediction is exact for many types of factorizations

Approach 4: Re-Ranking

PER	PER	-	-	LOC
PER	LOC	-	-	LOC
LOC	LOC	-	-	LOC
PER	PER	-	-	PER
PER	PER	PER	-	LOC
...	...	...	...	...
Jack	London	went	to	Paris

$$\hat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{A}(\mathcal{Y}^n)}{\operatorname{argmax}} \ \operatorname{score}(\mathbf{x}, \mathbf{y})$$

- Scoring of full inputs and outputs: very expressive!
- Relies on an active set $\mathcal{A}(\mathcal{Y}^n)$ of full outputs, enumerated exhaustively
- A base model is used to select active set
  - The base model follows one of the previous approaches

Sequence Prediction: Summary of Approaches

	input-output representation	exact prediction?
label classifiers	only individual labels	yes
transition-based	full history of decisons	no (greedy, beam search)
factored	label factors	yes
re-ranking	full	limited to active set

take home message 1: expressivity-tractability trade-off

take home message 2: always pick the simplest approach that suits the task at hand

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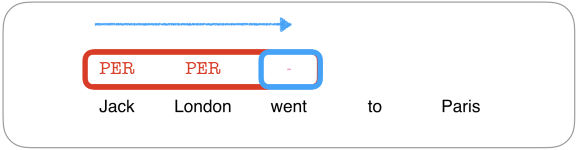
Greedy Sequence Prediction



- Run a greedy classifier left-to-right:
  - For $t = 1 \ldots n$:

$$\hat{y}_t = \underset{l \in \{\text{LOC}, \text{PER}, \text{-}\}}{\operatorname{argmax}} \operatorname{score}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1})$$

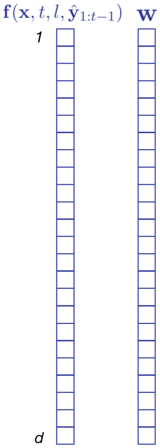
- What is the form of $\operatorname{score}(\mathbf{x}, t, l, \hat{y}_{1:t-1})$?
  - We focus on linear scoring functions: $\operatorname{score}(\mathbf{x}, t, l, \hat{y}_{1:t-1}) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, t, l, \hat{y}_{1:t-1})$

Representations in Greedy Sequence Prediction

- In linear greedy sequence prediction, at time t

$$\text{score}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1}) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1})$$

- $\mathbf{w} \in \mathbb{R}^d$ is a parameter vector, to be learned
- $\mathbf{f}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1}) \in \mathbb{R}^d$ is a feature vector
- Goal: guess the correct l at position t
- How to construct $\mathbf{f}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1})$?
  - New trend: representation learning
  - Old school: manually with feature templates



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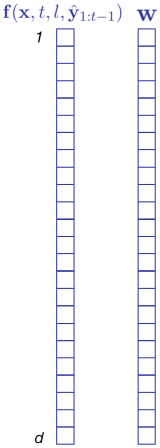
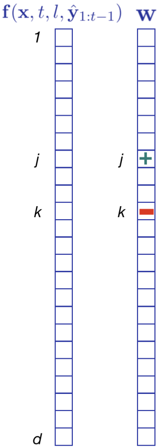


Indicator Features for One Label Only

- $\mathbf{f}(\mathbf{x}, t, l)$ is a vector of d features representing label l for x_t
- What's in a feature $\mathbf{f}_j(\mathbf{x}, t, l)$?
  - Anything we can compute using $\mathbf{x}$ and t and l
  - Anything that indicates whether l is (not) a good label for x_t
- Indicator features: binary-valued features looking at:
  - a simple pattern of $\mathbf{x}$ and target position t
  - and the candidate label l for position t

$$\mathbf{f}_j(\mathbf{x}, t, l) = \begin{cases} 1 & \text{if } x_t = \text{London and } l = \text{LOC} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{f}_k(\mathbf{x}, t, l) = \begin{cases} 1 & \text{if } x_{t+1} = \text{went and } l = \text{LOC} \\ 0 & \text{otherwise} \end{cases}$$



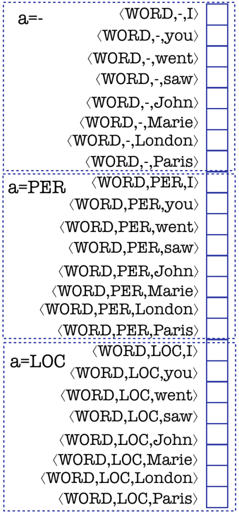
- Indicator features produce sparse feature vectors

Feature Templates

- Feature templates generate many indicator features
- A feature template is identified by a type, and a number of values
  - Example: template WORD indicates the current word

$$\mathbf{f}_{\langle \text{WORD}, a, w \rangle}(\mathbf{x}, t, l) = \begin{cases} 1 & \text{if } x_t = w \text{ and } l = a \\ 0 & \text{otherwise} \end{cases}$$

  - A feature of this type is identified by the tuple $\langle \text{WORD}, a, w \rangle$
  - Generates a feature for every label $a \in \mathcal{Y}$ and every word w
- Feature vectors and weight vectors are indexed by feature tuples



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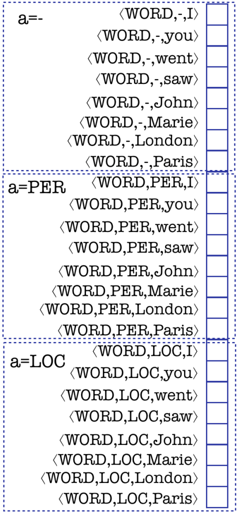
- Feature vectors and weight vectors are indexed by feature tuples
- In feature-based models:
  - Define feature templates manually
  - Instantiate the templates on every set of values in the training data $\rightarrow$ generates a very high-dimensional feature space
  - Define parameter vector $\mathbf{w}$ indexed by such feature tuples
  - Let the learning algorithm choose the relevant features

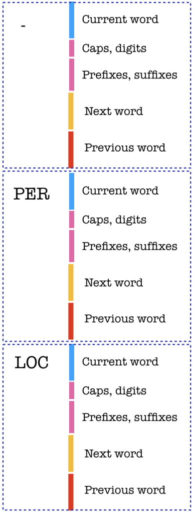


More Features for NE Recognition

PER
Jack London went to Paris

In practice, construct $\mathbf{f}(\mathbf{x}, t, l)$ by ...

- Define a number of simple patterns of $\mathbf{x}$ and t
  - current word x_t
  - is x_t capitalized?
  - x_t has digits?
  - prefixes/suffixes of size 1, 2, 3, ...
  - is x_t a known location?
  - is x_t a known person?
  - next word
  - previous word
  - current and next words together
  - other combinations
- Define feature templates by combining patterns with labels l
- Generate actual features by instantiating templates on training data



Feature Templates in Greedy Sequence Prediction

$\mathbf{y}$	PER	PER	-		
$\mathbf{x}$	Jack	London	went	to	Paris

- $\mathbf{f}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1})$ has access to all preceding labels
- Example: A template for word + current label + previous label:

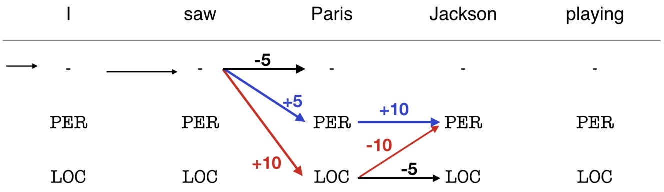
$$\mathbf{f}_{\langle \text{WB}, a, b, w \rangle}(\mathbf{x}, t, l, \hat{\mathbf{y}}_{1:t-1}) = \begin{cases} 1 & \text{if } x_t = w \text{ and} \\ & \quad \hat{\mathbf{y}}_{t-1} = a \text{ and } l = b \\ 0 & \text{otherwise} \end{cases}$$

- In practice:
  - Preceeding labels next to t
  - Bag-of-labels in $\hat{\mathbf{y}}_{1:t-1}$
  - Combinations with other features
- Neural networks automatically induce "good" features out of $\mathbf{x}$ and $\hat{\mathbf{y}}_{1:t-1}$

Transition Systems (general form)

- Given an input $\mathbf{x}$, a transition system defines:
  - A set of states $\mathcal{S}(\mathbf{x})$
  - An initial state $s_0 \in \mathcal{S}(\mathbf{x})$, and a set of final states $S_\infty \subseteq \mathcal{S}(\mathbf{x})$
  - A set of allowed actions $\mathcal{A}(s, \mathbf{x})$ for all $s \in \mathcal{S}(\mathbf{x})$
  - A transition function $\text{transition} : s \times a \rightarrow s'$
  - A scoring function: $\text{score} : \mathbf{x} \times s \times a \rightarrow \mathbb{R}$
- To predict output $\mathbf{y}$ from input $\mathbf{x}$:
  - $s = s_0$
  - while $s \notin S_\infty$:
    - $a = \text{argmax}_{a \in \mathcal{A}(s,\mathbf{x})} \text{score}(\mathbf{x}, s, a)$
    - $s = \text{transition}(s, a)$
  - extract $\mathbf{y}$ from s
- Simple, very fast and expressive! Very popular in NLP:
  - Greedy sequence prediction (one label at a time, left-to-right or right-to-left)
  - Shift-reduce parsing (more later)
  - Word segmentation, machine translation, . . .

Greedy Predictions are not Optimal, even with Beam Search



- Greedy sequence predictions can not undo decisions at a later stage
- Sometimes the model is right at a global scope, but not at each greedy step!
- Solution: Beam Search
  - General *local search* method
  - Maintains several good hypotheses, instead of just the best one
  - Many strategies, sometimes specific to the task and transition system
  - Empirically, it often improves over greedy search

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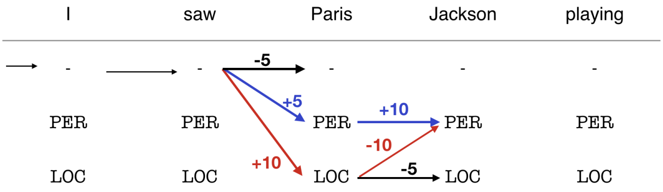

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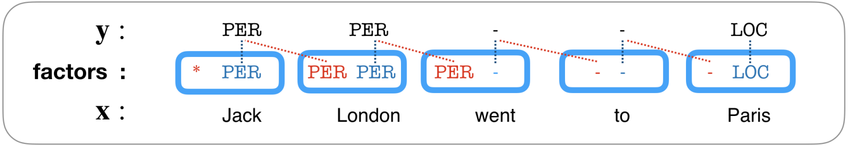
Factored Sequence Predictors



$$\hat{\mathbf{y}} = \underset{\mathbf{y} \in \mathcal{Y}^n}{\operatorname{argmax}} \sum_{i=1}^{n} \operatorname{score}(\mathbf{x}, i, y_{i-1}, y_i)$$

Next questions:

- What is the form of $\operatorname{score}(\mathbf{x}, i, a, b)$?
  We will use linear scoring functions: $\operatorname{score}(\mathbf{x}, i, a, b) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)$
- There are exponentially-many sequences $\mathbf{y}$ for a given $\mathbf{x}$,
  how do we solve the argmax problem?

Representations Factored at Bigrams

$\mathbf{y}$:	PER	PER	-	-	LOC
$\mathbf{x}$:	Jack	London	went	to	Paris

- $\text{score}(\mathbf{x}, i, a, b) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, a, b)$
- $\mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
  - A d-dimensional feature vector of a label bigram at i
  - Each dimension is typically a boolean indicator (0 or 1)
- $\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$
  - A d-dimensional feature vector of the entire $\mathbf{y}$
  - Aggregated representation by summing bigram feature vectors
  - Each dimension is now a count of a feature pattern

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Linear Factored Sequence Prediction

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) \qquad \text{where} \qquad \mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Note the linearity of the expression:

$$\begin{aligned}
\text{score}(\mathbf{x}, \mathbf{y}) &= \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) \\
&= \mathbf{w} \cdot \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) \\
&= \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) \\
&= \sum_{i=1}^{n} \text{score}(\mathbf{x}, i, y_{i-1}, y_i)
\end{aligned}$$

Predicting with Factored Sequence Models

- Assume we have a score function $\text{score}(\mathbf{x}, i, a, b)$
- Given $\mathbf{x}_{1:n}$ find:

$$\underset{\mathbf{y} \in \mathcal{Y}^n}{\text{argmax}} \sum_{i=1}^{n} \text{score}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Use the Viterbi algorithm, takes $O(n|\mathcal{Y}|^2)$
- Notational change: since $\mathbf{x}_{1:n}$ is fixed we will use

$$s(i, a, b) = \text{score}(\mathbf{x}, i, a, b)$$

Outline

Viterbi for Factored Sequence Models

- Given scores $s(i, a, b)$ for each position i and output bigram a, b, find:

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \sum_{i=1}^{n} s(i, y_{i-1}, y_i)$$

- Intuition: consider this example $\mathbf{x}$ and two alternative solutions $\mathbf{y}$ and $\mathbf{y}'$:

	1	2	3	4	5	6	7	8
$\mathbf{x}$	Jack	London	went	to	Paris	before	visiting	Lisbon
$\mathbf{y}$	PER	LOC	-	-	LOC	-	-	LOC
$\mathbf{y}'$	PER	PER	-	-	LOC	-	-	LOC

- What is the score of y' relative to the score of y?

$$s(\mathbf{x}, \mathbf{y}') = s(\mathbf{x}, \mathbf{y}) \quad + \qquad\qquad - $$
$$+ \qquad\qquad -$$

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$\mathbf{y}$	PER	LOC	-	-	LOC	-	-	LOC
$\mathbf{y}'$	PER	PER	-	-	LOC	-	-	LOC

- What is the score of y' relative to the score of y?

$$\begin{aligned} s(\mathbf{x}, \mathbf{y}') &= s(\mathbf{x}, \mathbf{y}) \quad + s(2, \text{PER}, \text{PER}) - s(2, \text{PER}, \text{LOC}) \\ &\qquad\qquad\quad + s(3, \text{PER}, \text{-}) - s(3, \text{LOC}, \text{-}) \end{aligned}$$

output sequences that share bigrams also share their scores

Viterbi recurrence

- Viterbi is a dynamic programming algorithm that uses the following recurrence
- Assume that, for a certain position i and each label $l \in \mathcal{Y}$, we have the best sub-sequence from positions 1 to i ending with label l:

$1 \quad \cdots \quad i \quad i+1$

best subsequence with $y_i = \text{PER}$

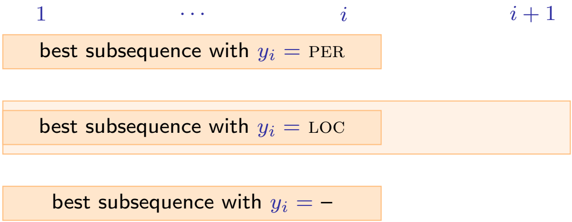
best subsequence with $y_i = \text{LOC}$

best subsequence with $y_i = -$

- What is the best sequence up to position $i+1$ with $y_{i+1} = \text{LOC}$?

Viterbi recurrence

- Viterbi is a dynamic programming algorithm that uses the following recurrence
- Assume that, for a certain position i and each label $l \in \mathcal{Y}$, we have the best sub-sequence from positions 1 to i ending with label l:



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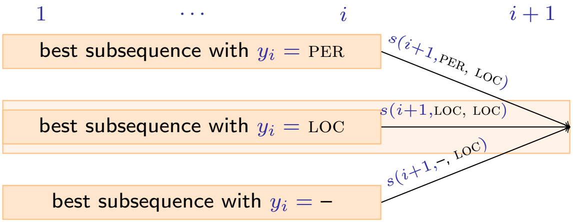

- What is the best sequence up to position $i+1$ with $y_{i+1} = \text{LOC}$?

Viterbi for Factored Sequence Models

$$\hat{\mathbf{y}} = \text{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \sum_{i=1}^{n} s(i, y_{i-1}, y_i)$$

- **Definition:** score of optimal sequence for $\mathbf{x}_{1:i}$ ending with $a \in \mathcal{Y}$

$$\delta(i, a) = \max_{\mathbf{y} \in \mathcal{Y}^i : y_i = a} \sum_{j=1}^{i} s(j, y_{j-1}, y_j)$$

- Use the following recursions, for all $a \in \mathcal{Y}$, for $i = 2 \dots n$:

$$\begin{aligned} \delta(1, a) &= s(1, y_0 = \text{NULL}, a) \\ \delta(i, a) &= \max_{b \in \mathcal{Y}} \delta(i-1, b) + s(i, b, a) \end{aligned}$$

- The optimal score for $\mathbf{x}$ is $\max_{a \in \mathcal{Y}} \delta(n, a)$
- The optimal sequence $\hat{\mathbf{y}}$ can be recovered through *back-pointers*
- Cost: $O(n|\mathcal{Y}|^2)$

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- The optimal score for $\mathbf{x}$ is $\max_{a \in \mathcal{Y}} \delta(n, a)$
- The optimal sequence $\hat{\mathbf{y}}$ can be recovered through *back-pointers*
- Homework: rewrite the Viterbi equations such that the algorithm proceeds right-to-left. Observe that the factored model remains the same (i.e. it is not a directional model)

Variations of Viterbi

- Sparse Viterbi
  - Only a few labels in $\mathcal{Y}$ apply to a position
  - Only a few label bigrams are possible
  - A sparse implementation cuts the $O(|\mathcal{Y}|^2)$ factor
- Higher-order Viterbi: factorize at trigrams instead of bigrams
  - Cost $O(n|\mathcal{Y}|^3)$
  - Very common in POS tagging (using sparse Viterbi to cut the $O(|\mathcal{Y}|^3)$ cost factor)
- k-best Viterbi: return the best k sequences (not just the single best)
  - Used in re-ranking approaches and some loss functions
- Forward-Backward: Viterbi for sum-product computations (instead of max-sum)

Forward-Backward Max-Sum Computations

- The Viterbi algorithm solves a max-sum recurrence

$$\max_{\mathbf{y} \in \mathcal{Y}^n} \sum_{i=1}^{n} s(i, y_{i-1}, y_i)$$

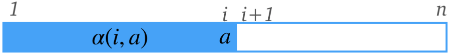
- The sum-product recurrence is also very useful (more later)

$$\sum_{\mathbf{y} \in \mathcal{Y}^n} \prod_{i=1}^{n} s(i, y_{i-1}, y_i)$$

- The same style of dynamic programming works

Forward Algorithm

$$\sum_{\mathbf{y} \in \mathcal{Y}^n} \prod_{i=1}^{n} s(i, y_{i-1}, y_i)$$



- **Definition:** forward quantities

$$\alpha(i,a) \quad = \quad \sum_{\mathbf{y}_{1:i} \in \mathcal{Y}^i : y_i = a} \prod_{j=1}^{i} s(j, y_{j-1}, y_j)$$

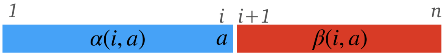
- Use the following recursions, for all $a \in \mathcal{Y}$, for $i = 2 \ldots n$:

$$\begin{aligned} \alpha(i,a) &= s(1, y_0 = \text{NULL}, a) \\ \alpha(i,a) &= \sum_{b \in \mathcal{Y}} \alpha(i-1, b) * s(i, b, a) \end{aligned}$$

- The total sum-product is $\sum_a \alpha(n,a)$
- Like Viterbi, the forward algorithm runs in $O(n|\mathcal{Y}|^2)$

Backward Algorithm

$$\sum_{\mathbf{y} \in \mathcal{Y}^n} \prod_{i=1}^{n} s(i, y_{i-1}, y_i)$$



- **Definition:** backward quantities

$$\beta(i, a) = \sum_{\mathbf{y}_{i:n} \in \mathcal{Y}^{(n-i+1)} : y_i = a} \prod_{j=i+1}^{n} s(j, y_{j-1}, y_j)$$

- Now the recursions run *backwards*! For all $a \in \mathcal{Y}$, for $i = n - 1 \ldots 1$:

$$\begin{aligned} \beta(n, a) &= 1 \\ \beta(i, a) &= \sum_{b \in \mathcal{Y}} s(i, a, b) * \beta(i + 1, b) \end{aligned}$$

- The total sum-product is $\sum_a s(1, y_0 = \text{NULL}, a) * \beta(1, a)$
- Like Viterbi and forward algorithms, the backward algorithm runs in $O(n|\mathcal{Y}|^2)$

Outline

Log-linear Models for Sequence Prediction

- Model the conditional distribution:

$$\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})}$$

  where
  - $\mathbf{f}(\mathbf{x}, \mathbf{y})$ represents $\mathbf{x}$ and $\mathbf{y}$ with d features
  - $\mathbf{w} \in \mathbb{R}^d$ are the parameters of the model
  - $Z(\mathbf{x}; \mathbf{w})$ is a normalizer called the *partition function*

$$Z(\mathbf{x}; \mathbf{w}) = \sum_{\mathbf{z} \in \mathcal{Y}^*} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}$$

- To predict the best sequence

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \Pr(\mathbf{y} | \mathbf{x})$$

Log-linear Models: Name

- Let's take the $\log$ of the conditional probability:

$$
\begin{aligned}
\log \Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) &= \log \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})} \\
&= \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) - \log \sum_{y} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\} \\
&= \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y}) - \log Z(\mathbf{x}; \mathbf{w})
\end{aligned}
$$

- Partition function: $Z(\mathbf{x}; \mathbf{w}) = \sum_{\mathbf{z}} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})\}$
- $\log Z(\mathbf{x}; \mathbf{w})$ is a constant for a fixed $\mathbf{x}$
- In the $\log$ space, computations are linear,
  i.e., we model log-probabilities using a linear predictor

Making Predictions with Log-Linear Models

- For tractability, assume $\mathbf{f}(\mathbf{x}, \mathbf{y})$ decomposes into bigrams:

$$\mathbf{f}(\mathbf{x}_{1:n}, \mathbf{y}_{1:n}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Given $\mathbf{w}$, given $\mathbf{x}_{1:n}$, find:

$$\begin{aligned}
\underset{\mathbf{y}_{1:n}}{\operatorname{argmax}} \Pr(\mathbf{y}_{1:n} | \mathbf{x}_{1:n}; \mathbf{w}) &= \underset{\mathbf{y}}{\operatorname{amax}} \frac{\exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x}; \mathbf{w})} \\
&= \underset{\mathbf{y}}{\operatorname{amax}} \exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\} \\
&= \underset{\mathbf{y}}{\operatorname{amax}} \sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)
\end{aligned}$$

- We can use the Viterbi algorithm

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$$\begin{aligned}
\operatorname*{argmax}_{\mathbf{y}_{1:n}} \Pr(\mathbf{y}_{1:n} | \mathbf{x}_{1:n}; \mathbf{w}) &= \operatorname*{amax}_{\mathbf{y}} \frac{\exp\left\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\right\}}{Z(\mathbf{x}; \mathbf{w})} \\
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\end{aligned}$$

- We can use the Viterbi algorithm

Probability of an Output Sequence given an Input Sequence

- Given $\mathbf{x}$ and $\mathbf{y}$, compute $\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})}$
- To compute $Z(\mathbf{x}; \mathbf{w})$ we need to sum over $\mathcal{Y}^n$!
- But with some algebraic massaging: (let $s(i, y_{i-1}, y_i) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$)

$$\begin{aligned} Z(\mathbf{x}; \mathbf{w}) &= \sum_{\mathbf{y}} \exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\} \\ &= \sum_{\mathbf{y}} \exp\left\{\sum_{i=1}^{n} s(i, y_{i-1}, y_i)\right\} \\ &= \sum_{\mathbf{y}} \prod_{i=1}^{n} \exp\left\{s(i, y_{i-1}, y_i)\right\} \end{aligned}$$

- $Z(\mathbf{x}; \mathbf{w})$ is a sum-product computation: forward algorithm (with exponentiated scores)!
  - $Z(\mathbf{x}; \mathbf{w}) = \sum_a \alpha(n, a)$

Marginal Probability of a Single Label

PER

I saw Paris Jackson playing

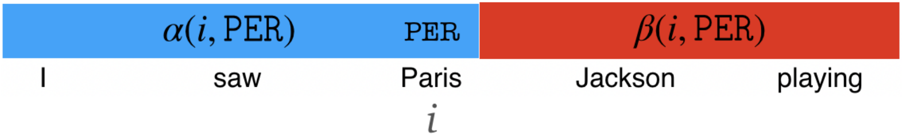
i

- What's the probability that token i has label a?
- We need to compute the marginal distribution of y_i:

$$\begin{aligned}\mu_i(a) = \Pr(y_i = a|\mathbf{x};\mathbf{w}) &= \sum_{\mathbf{y}\in\mathcal{Y}^n : y_i = a} \Pr(\mathbf{y}|\mathbf{x};\mathbf{w}) \\ &= (\text{algebraic massaging}) \\ &= \frac{\alpha(i,a) * \beta(i,a)}{Z(\mathbf{x};\mathbf{w})}\end{aligned}$$

- Use forward-backward (using exponentiated scores)
  - Recall that $Z(\mathbf{x};\mathbf{w}) = \sum_l \alpha(n,l)$

Marginal Probability of a Single Label



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- Use forward-backward (using exponentiated scores)
  - Recall that $Z(\mathbf{x}; \mathbf{w}) = \sum_l \alpha(n, l)$

Marginal Probability of a Label Bigram

		PER	PER	
I	saw	Paris	Jackson	playing
		i-1	*i*	

- What's the probability that token $i-1$ has label a and token i has label b?
- We need to compute the marginal distribution of label bigrams at position i:

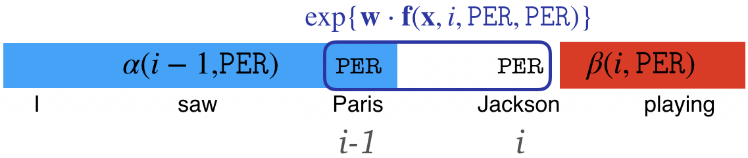
$$\mu_i(a,b) = \Pr(y_{i-1}=a, y_i=b|\mathbf{x};\mathbf{w}) = \sum_{\mathbf{y}\in\mathcal{Y}^n : y_{i-1}=a, y_i=b} \Pr(\mathbf{y}|\mathbf{x};\mathbf{w})$$

$$= \text{(algebraic massaging)}$$

$$= \frac{\alpha(i-1,a) * \exp\{\mathbf{w}\cdot\mathbf{f}(\mathbf{x},i,a,b)\} * \beta(i,b)}{Z(\mathbf{x};\mathbf{w})}$$

- Again forward-backward (using exponentiated scores)
  - Recall that $Z(\mathbf{x};\mathbf{w}) = \sum_l \alpha(n,l)$

Marginal Probability of a Label Bigram



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$$
\begin{aligned}
\mu_i(a,b) = \Pr(y_{i-1}=a, y_i=b \,|\, \mathbf{x};\mathbf{w}) &= \sum_{\mathbf{y}\in\mathcal{Y}^n : y_{i-1}=a, y_i=b} \Pr(\mathbf{y}|\mathbf{x};\mathbf{w}) \\
&= \text{(algebraic massaging)} \\
&= \frac{\alpha(i-1,a) * \exp\{\mathbf{w}\cdot\mathbf{f}(\mathbf{x},i,a,b)\} * \beta(i,b)}{Z(\mathbf{x};\mathbf{w})}
\end{aligned}
$$

- Again forward-backward (using exponentiated scores)
  - Recall that $Z(\mathbf{x};\mathbf{w}) = \sum_l \alpha(n,l)$

Linear Factored Sequence Prediction

$$\underset{\mathbf{y} \in \mathcal{Y}^n}{\operatorname{argmax}} \, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})$$

- Factored representation, e.g. based on bigrams

$$\mathbf{f}(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Flexible, arbitrary features of full $\mathbf{x}$ and the factors
- Efficient prediction using Viterbi
- In probabilistic models, efficient computation of marginals using Forward-Backward
- Next, learning $\mathbf{w}$:
  - The Structured Perceptron
  - Probabilistic log-linear models:
    - Local learning, *a.k.a.* Maximum-Entropy Markov Models
    - Global learning, *a.k.a.* Conditional Random Fields

Outline

The Structured Perceptron

Collins (2002)

- Set $\mathbf{w} = \mathbf{0}$
- For $t = 1 \ldots T$
  - For each training example $(\mathbf{x}, \mathbf{y})$
    1. Compute $\mathbf{z} = \mathrm{argmax}_{\mathbf{z}} \, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
    2. If $\mathbf{z} \neq \mathbf{y}$

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z})$$

- Return $\mathbf{w}$

The Structured Perceptron + Averaging

Freund and Schapire (1999); Collins (2002)

- Set $\mathbf{w} = \mathbf{0}$, $\mathbf{w}^a = \mathbf{0}$
- For $t = 1 \ldots T$
  - For each training example $(\mathbf{x}, \mathbf{y})$
    1. Compute $\mathbf{z} = \text{argmax}_{\mathbf{z}}\, \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{z})$
    2. If $\mathbf{z} \neq \mathbf{y}$

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z})$$

    3. $\mathbf{w^a} = \mathbf{w^a} + \mathbf{w}$
- Return $\mathbf{w^a}$

Perceptron Updates: Example

$\mathbf{y}$	PER	PER	-	-	LOC
$\mathbf{z}$	PER	LOC	-	-	LOC
$\mathbf{x}$	Jack	London	went	to	Paris

- Let $\mathbf{y}$ be the correct output for $\mathbf{x}$.
- Say we predict $\mathbf{z}$ instead, under our current $\mathbf{w}$
- The update is:

$$\begin{aligned}
\mathbf{g} &= \mathbf{f}(\mathbf{x}, \mathbf{y}) - \mathbf{f}(\mathbf{x}, \mathbf{z}) \\
&= \sum_i \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i) - \sum_i \mathbf{f}(\mathbf{x}, i, z_{i-1}, z_i) \\
&= \mathbf{f}(\mathbf{x}, 2, \text{PER}, \text{PER}) - \mathbf{f}(\mathbf{x}, 2, \text{PER}, \text{LOC}) \\
&\quad + \mathbf{f}(\mathbf{x}, 3, \text{PER}, \text{-}) - \mathbf{f}(\mathbf{x}, 3, \text{LOC}, \text{-})
\end{aligned}$$

- Perceptron updates are typically very sparse

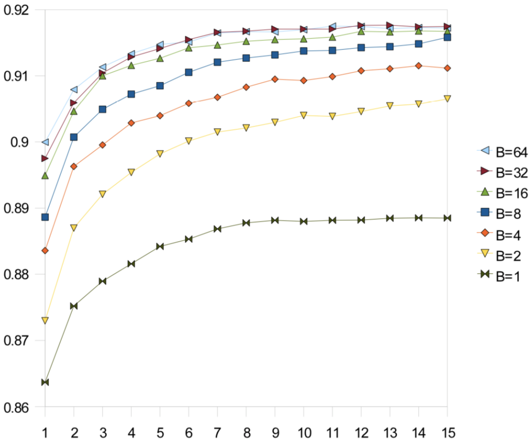
Properties of the Perceptron

- Online algorithm. Often much more efficient than "batch" algorithms
- If the data is separable, it will converge to parameter values with 0 errors
- Number of errors before convergence is related to a definition of *margin*. Can also relate margin to generalization properties
- In practice:
    1. Averaging improves performance a lot
    2. Typically reaches a good solution after only a few (say 5) iterations over the training set
    3. Often performs nearly as well as CRFs, or SVMs
- Structured Perceptron and Beam Search:
    - Transition systems can not recover the argmax solution
    - Structured Perceptron can use beam search instead (i.e. an approximation to argmax)
    - See Collins and Roark (2004); Zhang and Clark (2011); Huang et al. (2012)

Averaged Perceptron Convergence

Iteration	Accuracy
1	90.79
2	91.20
3	91.32
4	91.47
5	91.58
6	91.78
7	91.76
8	91.82
9	91.88
10	91.91
11	91.92
12	91.96
...	

results on validation set for a parsing task



perceptron with beam search
(Zhang and Clark, 2011)

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- To predict the best sequence

$$\operatorname*{argmax}_{\mathbf{y} \in \mathcal{Y}^n} \Pr(\mathbf{y} | \mathbf{x})$$

Parameter Estimation in Log-Linear Models

$$\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w}) = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, \mathbf{y})\}}{Z(\mathbf{x}; \mathbf{w})}$$

- Given training data

$$\left\{(\mathbf{x}^{(1)}, \mathbf{y}^{(1)}), (\mathbf{x}^{(2)}, \mathbf{y}^{(2)}), \ldots, (\mathbf{x}^{(m)}, \mathbf{y}^{(m)})\right\} \quad ,$$

- How to estimate $\mathbf{w}$?
- Define the conditional log-likelihood (or cross-entropy) of the data:

$$L(\mathbf{w}) = \sum_{k=1}^{m} \log \Pr(\mathbf{y}^{(k)} | \mathbf{x}^{(k)}; \mathbf{w})$$

- $L(\mathbf{w})$ measures how well $\mathbf{w}$ explains the data. A good value for $\mathbf{w}$ will give a high value for $\Pr(\mathbf{y}^{(k)}|\mathbf{x}^{(k)}; \mathbf{w})$ for all $k = 1 \ldots m$.
- We want $\mathbf{w}$ that maximizes $L(\mathbf{w})$

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Learning Log-Linear Models: Loss + Regularization

- Solve:

$$\mathbf{w}^* = \underset{\mathbf{w} \in \mathbb{R}^d}{\operatorname{argmin}} \overbrace{-L(\mathbf{w})}^{\text{Loss}} + \overbrace{\frac{\lambda}{2}||\mathbf{w}||^2}^{\text{Regularization}}$$

  where
  - The first term is the negative conditional log-likelihood
  - The second term is a regularization term, it penalizes solutions with large norm
  - $\lambda \in \mathbb{R}$ controls the trade-off between loss and regularization
- Convex optimization problem → gradient descent
- Two common losses based on log-likelihood that make learning tractable:
  - Local Loss (MEMM): assume that $\Pr(\mathbf{y} \mid \mathbf{x}; \mathbf{w})$ decomposes
  - Global Loss (CRF): assume that $\mathbf{f}(\mathbf{x}, \mathbf{y})$ decomposes

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Local Log-linear Loss (a.k.a. Maximum Entropy Markov Models)

McCallum, Freitag, and Pereira (2000)

- If we apply the chain rule:

$$\begin{aligned}\Pr(\mathbf{y}_{1:n} \mid \mathbf{x}_{1:n}) &= \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \Pr(\mathbf{y}_{2:n} \mid \mathbf{x}_{1:n}, y_1) \\ &= \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i \mid \mathbf{x}_{1:n}, \mathbf{y}_{1:i-1})\end{aligned}$$

- Markov assumption (the model becomes factored):

$$\Pr(y_i \mid \mathbf{x}_{1:n}, \mathbf{y}_{1:i-1}) = \Pr(y_i \mid \mathbf{x}_{1:n}, y_{i-1})$$

- Now we can write

$$\Pr(\mathbf{y}_{1:n} \mid \mathbf{x}_{1:n}) = \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i \mid \mathbf{x}_{1:n}, \mathbf{y}_{i-1})$$

Parameter Estimation with Local Log-Linear Markov Models

$$\Pr(y_{1:n} \mid \mathbf{x}_{1:n}) \quad = \quad \Pr(y_1 \mid \mathbf{x}_{1:n}) \times \prod_{i=2}^{n} \Pr(y_i | \mathbf{x}_{1:n}, i, y_{i-1})$$

- The log-linear model is normalized locally (i.e. at each position):

$$\Pr(y \mid \mathbf{x}, i, y') = \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y', y)\}}{Z(\mathbf{x}, i, y')}$$

- The log-likelihood is also local :

$$L(\mathbf{w}) = \sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}} \log \Pr(\mathbf{y}_i^{(k)} | \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)})$$

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_j} = \frac{1}{m} \sum_{k=1}^{m} \sum_{i=1}^{n^{(k)}} \left[\overbrace{\mathbf{f}_j(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, \mathbf{y}_i^{(k)})}^{\text{observed}} - \overbrace{\sum_{y \in \mathcal{Y}} \Pr(\mathbf{y} | \mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y) \; \mathbf{f}_j(\mathbf{x}^{(k)}, i, \mathbf{y}_{i-1}^{(k)}, y)}^{\text{expected}} \right]$$

Conditional Random Fields

Lafferty, McCallum, and Pereira (2001)

- Log-linear model of the conditional distribution:

$$\Pr(\mathbf{y}|\mathbf{x};\mathbf{w}) = \frac{\exp\{\mathbf{w}\cdot\mathbf{f}(\mathbf{x},\mathbf{y})\}}{Z(\mathbf{x})}$$

  where
  - $\mathbf{x}$ and $\mathbf{y}$ are input and output sequences
  - $\mathbf{f}(\mathbf{x},\mathbf{y})$ is a feature vector of $\mathbf{x}$ and $\mathbf{y}$ that decomposes into factors
  - $\mathbf{w}$ are model parameters
- To predict the best sequence

$$\hat{\mathbf{y}} = \operatorname*{argmax}_{\mathbf{y}\in\mathcal{Y}^*} \Pr(\mathbf{y}|\mathbf{x})$$

- Log-Likelihood at the global (sequence) level:

$$L(\mathbf{w}) = \sum_{k=1}^{m} \log \Pr(\mathbf{y}^{(k)}|\mathbf{x}^{(k)};\mathbf{w})$$

Computing the Gradient in CRFs

Consider a parameter $\mathbf{w}_j$ and its associated feature $\mathbf{f}_j$:

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}_j} = \frac{1}{m} \sum_{k=1}^{m} \left[\overbrace{\mathbf{f}_j(\mathbf{x}^{(k)}, \mathbf{y}^{(k)})}^{\text{observed}} - \overbrace{\sum_{\mathbf{y} \in \mathcal{Y}^*} \Pr(\mathbf{y} | \mathbf{x}^{(k)}; \mathbf{w}) \ \mathbf{f}_j(\mathbf{x}^{(k)}, \mathbf{y})}^{\text{expected}} \right]$$

where

$$\mathbf{f}_j(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} \mathbf{f}_j(\mathbf{x}, i, y_{i-1}, y_i)$$

- First term: observed value of $\mathbf{f}_j$ in training examples
- Second term: expected value of $\mathbf{f}_j$ under current $\mathbf{w}$
  they require summing over all sequences $\mathbf{y} \in \mathcal{Y}^n$

Computing the Gradient in CRFs

- For an example $(\mathbf{x}^{(k)}, \mathbf{y}^{(k)})$:

$$\sum_{\mathbf{y} \in \mathcal{Y}^n} \Pr(\mathbf{y}|\mathbf{x}^{(k)}; \mathbf{w}) \sum_{i=1}^{n} \mathbf{f}_j(\mathbf{x}^{(k)}, i, y_{i-1}, y_i) = \sum_{i=1}^{n} \sum_{a,b \in \mathcal{Y}} \mu_i^k(a,b) \mathbf{f}_j(\mathbf{x}^{(k)}, i, a, b)$$

- $\mu_i^k(a,b)$ is the marginal probability of having labels (a,b) at position i:

$$\mu_i^k(a,b) = \Pr(\langle i, a, b \rangle \mid \mathbf{x}^{(k)}; \mathbf{w}) = \sum_{\mathbf{y} \in \mathcal{Y}^n \;:\; y_{i-1}=a,\; y_i=b} \Pr(\mathbf{y}|\mathbf{x}^{(k)}; \mathbf{w})$$

- The quantities μ_i^k can be computed efficiently in $O(nL^2)$ using the forward-backward algorithm

CRFs: summary so far

- Log-linear models for sequence prediction, $\Pr(\mathbf{y}|\mathbf{x};\mathbf{w})$
- Computations factorize on label bigrams
- Model form:

$$\underset{\mathbf{y}\in\mathcal{Y}^*}{\operatorname{argmax}} \sum_i \mathbf{w}\cdot\mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Prediction: uses Viterbi
- Parameter estimation:
  - Gradient-based methods, in practice L-BFGS or SGD
  - Computation of gradient uses forward-backward

CRFs: summary so far

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- Prediction: uses Viterbi
- Parameter estimation:
  - Gradient-based methods, in practice L-BFGS or SGD
  - Computation of gradient uses forward-backward

- Next Question: Local or Global loss?

Local vs. Global Log-linear Losses

Local Loss: $\Pr(\mathbf{y} \mid \mathbf{x}) = \prod_{i=1}^{n} \frac{\exp\{\mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\}}{Z(\mathbf{x}, i, y_{i-1}; \mathbf{w})}$

CRFs: $\Pr(\mathbf{y} \mid \mathbf{x}) = \frac{\exp\{\sum_{i=1}^{n} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)\}}{Z(\mathbf{x})}$

- Both exploit the same factorization, i.e. same features
- Same computations to compute $\operatorname{argmax}_{\mathbf{y}} \Pr(\mathbf{y} \mid \mathbf{x})$
- Local loss is locally normalized; CRFs globally normalized
  - Local loss assumes that $\Pr(y_i \mid x_{1:n}, y_{1:i-1}) = \Pr(y_i \mid x_{1:n}, y_{i-1})$
  - Leads to "Label Bias Problem" (Lafferty et al., 2001; Andor et al., 2016)
- Local loss is cheaper to train (reduces to multiclass MaxEnt learning)
- CRFs are easier to extend to other structures

Learning Structure Predictors: summary so far

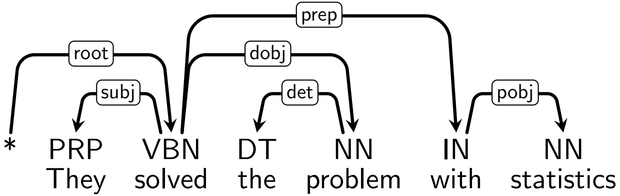
- Linear models for sequence prediction

$$\underset{\mathbf{y}\in\mathcal{Y}^*}{\operatorname{argmax}} \sum_i \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, y_{i-1}, y_i)$$

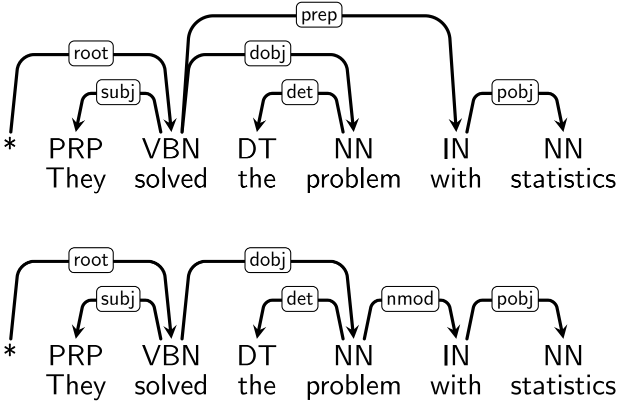
- Computations factorize on label bigrams
  - Decoding: using Viterbi
  - Marginals: using forward-backward
- Parameter estimation:
  - Perceptron, Log-likelihood, SVMs
  - Extensions from classification to the structured case
  - Optimization methods:
    - Stochastic (sub)gradient methods (LeCun et al., 1998; Shalev-Shwartz et al., 2011)
    - Exponentiated Gradient (Collins et al., 2008)
    - SVM Struct (Tsochantaridis et al., 2005)
    - Structured MIRA (Crammer et al., 2005)

Outline

Dependency Parsing

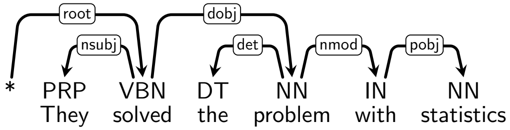


Dependency Parsing



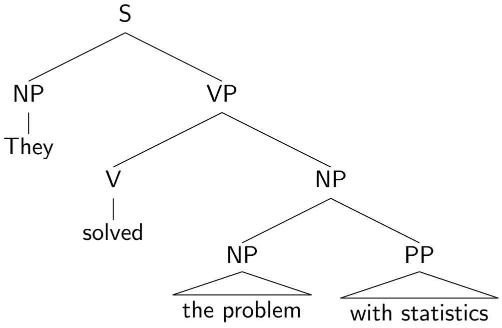
Theories of Syntactic Structure

Dependency Trees



- Main element: dependency
- Focus on relations between words

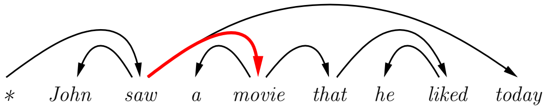
Constituent Trees



- Main element: constituents (or phrases, or bracketings)
- Constituents = abstract linguistic units
- Results in nested trees

Dependency Parsing: Arc-factored models

McDonald, Pereira, Ribarov, and Hajič (2005)



- Parse trees decompose into single dependencies $\langle h, m \rangle$

$$\underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h,m \rangle \in y} \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, h, m)$$

- Each arc or dependency (h, m) is scored independently of each other
- Some features: $\mathbf{f}_1(\mathbf{x}, h, m) = [\text{ "saw" } \rightarrow \text{"movie" }]$
  $\mathbf{f}_2(\mathbf{x}, h, m) = [\text{ distance} = +2 \ ]$
- Tractable inference algorithms exist

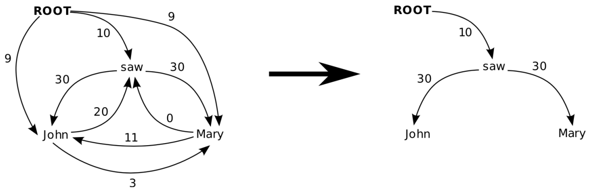
MST Parsing for Arc-factored models

McDonald, Pereira, Ribarov, and Hajič (2005)

- Parsing problem, given a sentenc $\mathbf{x}$:

$$\underset{\mathbf{y}\in\mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h,m\rangle\in\mathbf{y}} \operatorname{score}(\mathbf{x}, h, m)$$

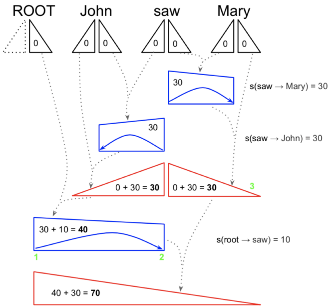
- Can be formulated as a directed Maximum Spanning Tree (MST) problem:



- The Chu-Liu-Edmonds algorithm finds the optimal tree in $O(n^2)$

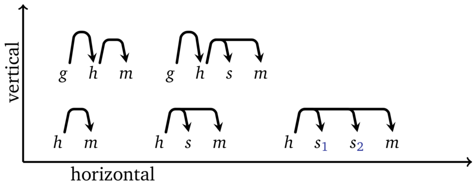
The Eisner Algorithm for Arc-factored models

Eisner (1996); McDonald and Pereira (2006); Carreras (2007); Koo and Collins (2010)



(illustration by Joakim Nivre)

Extension to higher-order parsing:



- First-order $O(n^3)$
- Second-order:
  - Horizontal $O(n^3)$ (McDonald and Pereira, 2006)
  - Vertical $O(n^4)$ (Carreras, 2007)
- Third-order $O(n^4)$ (Koo and Collins, 2010)

- The Eisner (1996) algorithm is a variant of CKY specific to non-crossing dep trees
- Finds optimal tree in $O(n^3)$

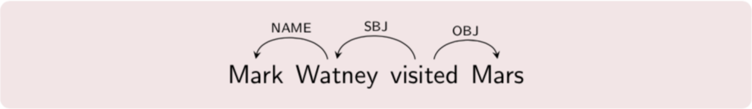
Transition-based Parsing: Nivre's Arc-Standard System

Nivre (2008)

- State:
  - Buffer: list of upcoming words to be parsed
  - Stack: stack of subtrees that are already parsed
- Parsing actions:
  - Shift: shift next word in the buffer to the task
  - Left-arc (l): add a left arc between the two top subtrees of the stack, with label l
  - Right-arc (l): add a right arc between the two top subtrees of the stack, with label l

- Parsing is linear in the sentence length, very fast! But prone to greedy mistakes!
- Parsing model: score a candidate action in the context of a state
  - Has access to the full sentence and the full history of actions

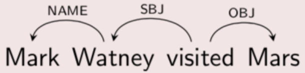
Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)



transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]

Mark Watney visited Mars

Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)



transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]

Mark Watney visited Mars

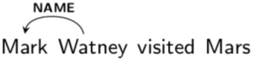
Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)

transition	**Stack**	**Buffer**
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]

Mark Watney visited Mars

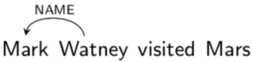
Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)

transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]
LA(NAME)	[Watney]	[visited, Mars]



Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)

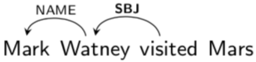
transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]
LA(NAME)	[Watney]	[visited, Mars]
SHIFT	[Watney, visited]	[Mars]



Arc-Standard Parsing: Example

(illustration by Miguel Ballesteros)

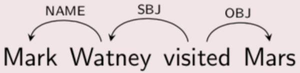
transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]
LA(NAME)	[Watney]	[visited, Mars]
SHIFT	[Watney, visited]	[Mars]
LA(SUBJ)	[visited]	[Mars]



Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)

transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]
LA(NAME)	[Watney]	[visited, Mars]
SHIFT	[Watney, visited]	[Mars]
LA(SUBJ)	[visited]	[Mars]
SHIFT	[visited, Mars]	[ ]

Arc-Standard Parsing: Example
(illustration by Miguel Ballesteros)



transition	Stack	Buffer
	[ ]	[Mark, Watney, visited, Mars]
SHIFT	[Mark]	[Watney, visited, Mars]
SHIFT	[Mark, Watney]	[visited, Mars]
LA(NAME)	[Watney]	[visited, Mars]
SHIFT	[Watney, visited]	[Mars]
LA(SUBJ)	[visited]	[Mars]
SHIFT	[visited, Mars]	[ ]
RA(OBJ)	[visited]	[ ]

Outline

Linear (Structured) Prediction

- Multiclass classification

$$\underset{\mathbf{y}\in\{1,\dots,L\}}{\operatorname{argmax}} \ \mathbf{w}\cdot\mathbf{f}(\mathbf{x},\mathbf{y})$$

- Sequence prediction (bigram factorization)

$$\underset{\mathbf{y}\in\mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \mathbf{w}\cdot\mathbf{f}(\mathbf{x},\mathbf{y}) = \underset{\mathbf{y}\in\mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_i \mathbf{w}\cdot\mathbf{f}(\mathbf{x},i,\mathbf{y}_{i-1},\mathbf{y}_i)$$

- Dependency parsing (arc-factored)

$$\underset{\mathbf{y}\in\mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \mathbf{w}\cdot\mathbf{f}(\mathbf{x},\mathbf{y}) = \underset{\mathbf{y}\in\mathcal{Y}(\mathbf{x})}{\operatorname{argmax}} \sum_{\langle h,m,l\rangle\in y} \mathbf{w}\cdot\mathbf{f}(\mathbf{x},h,m,l)$$

- Factored models: Applicable to other tasks and factorizations
- Alternative: transition systems (very fast and expressive, but prone to search errors)

Factored Sequence Prediction: from Linear to Non-linear

$$\text{score}(\mathbf{x}, \mathbf{y}) = \sum_i \text{s}(\mathbf{x}, i, y_{i-1}, y_i)$$

- Linear:

$$\text{s}(\mathbf{x}, i, y_{i-1}, y_i) = \mathbf{w} \cdot \mathbf{f}(\mathbf{x}, i, \mathbf{y}_{i-1}, \mathbf{y}_i)$$

- Non-linear, using a feed-forward neural network:

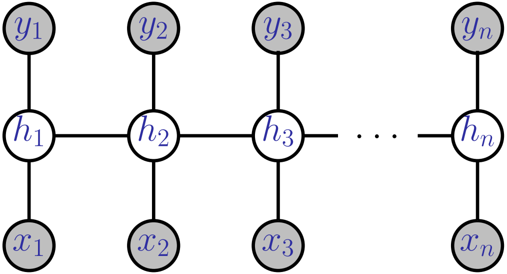
$$\text{s}(\mathbf{x}, i, y_{i-1}, y_i) = \mathbf{w} \cdot [e_{y_{i-1}, y_i} \otimes h(\mathbf{f}(\mathbf{x}, i))]$$

  where:

$$h(\mathbf{f}(\mathbf{x}, i)) = \sigma(W^2 \sigma(W^1 \sigma(W^0 \mathbf{f}(\mathbf{x}, i))))$$

- Remarks:
  - The non-linear model computes a hidden representation of the input
  - Still factored: Viterbi and Forward-Backward work
  - Parameter estimation becomes non-convex, use backpropagation

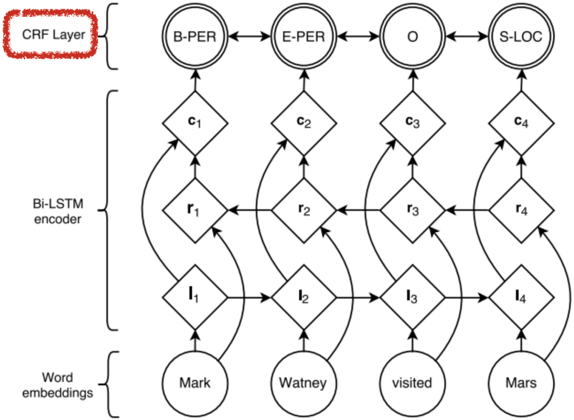
Recurrent Sequence Prediction



- Induction of hidden vectors (i.e. embeddings) that keep track of previous observations and predictions
- Making predictions is not tractable
  - In practice: greedy predictions or beam search
  - Making predictions was not tractable for transition systems either!
- Learning is non-convex, so what?
- Popular methods: RNN, LSTM, Spectral Models, ...

Neural Architectures for Named Entity Recognition

Guillaume Lample♠ Miguel Ballesteros♣♠
Sandeep Subramanian♠ Kazuya Kawakami♠ Chris Dyer♠

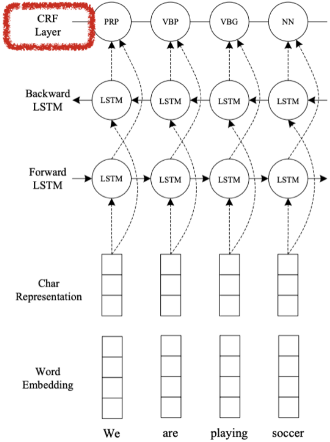


Model	F_1
Collobert et al. (2011)*	89.59
Lin and Wu (2009)	83.78
Lin and Wu (2009)*	90.90
Huang et al. (2015)*	90.10
Passos et al. (2014)	90.05
Passos et al. (2014)*	90.90
Luo et al. (2015)* + gaz	89.9
Luo et al. (2015)* + gaz + linking	**91.2**
Chiu and Nichols (2015)	90.69
Chiu and Nichols (2015)*	90.77
LSTM-CRF (no char)	90.20
LSTM-CRF	**90.94**
S-LSTM (no char)	87.96
S-LSTM	90.33

Table 1: English NER results (CoNLL-2003 test set).

End-to-end Sequence Labeling via Bi-directional LSTM-CNNs-CRF

Xuezhe Ma and **Eduard Hovy**



Model	POS		NER					
	Dev	Test	Dev			Test		
	Acc.	Acc.	Prec.	Recall	F1	Prec.	Recall	F1
BRNN	96.56	96.76	92.04	89.13	90.56	87.05	83.88	85.44
BLSTM	96.88	96.93	92.31	90.85	91.57	87.77	86.23	87.00
BLSTM-CNN	97.34	97.33	92.52	93.64	93.07	88.53	90.21	89.36
BRNN-CNN-CRF	97.46	97.55	94.85	94.63	94.74	91.35	91.06	91.21

Table 3: Performance of our model on both the development and test sets of the two tasks, together with three baseline systems.

Model	Acc.
Giménez and Màrquez (2004)	97.16
Toutanova et al. (2003)	97.27
Manning (2011)	97.28
Collobert et al. (2011)‡	97.29
Santos and Zadrozny (2014)‡	97.32
Shen et al. (2007)	97.33
Sun (2014)	97.36
Søgaard (2011)	97.50
This paper	**97.55**

Table 4: POS tagging accuracy of our model on test data from WSJ proportion of PTB, together

Model	F1
Chieu and Ng (2002)	88.31
Florian et al. (2003)	88.76
Ando and Zhang (2005)	89.31
Collobert et al. (2011)‡	89.59
Huang et al. (2015)‡	90.10
Chiu and Nichols (2015)‡	90.77
Ratinov and Roth (2009)	90.80
Lin and Wu (2009)	90.90
Passos et al. (2014)	90.90
Lample et al. (2016)‡	90.94
Luo et al. (2015)	91.20
This paper	**91.21**

Table 5: NER F1 score of our model on test data set from CoNLL-2003. For the purpose of com-

Thanks!

References I

Daniel Andor, Chris Alberti, David Weiss, Aliaksei Severyn, Alessandro Presta, Kuzman Ganchev, Slav Petrov, and Michael Collins. Globally normalized transition-based neural networks. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pages 2442–2452, Berlin, Germany, August 2016. Association for Computational Linguistics. URL http://www.aclweb.org/anthology/P16-1231.

Xavier Carreras. Experiments with a higher-order projective dependency parser. In *Proceedings of the 2007 Joint Conference on Empirical Methods in Natural Language Processing and Computational Natural Language Learning (EMNLP-CoNLL)*, 2007.

Michael Collins. Discriminative training methods for hidden markov models: Theory and experiments with perceptron algorithms. In *Proceedings of the ACL-02 conference on Empirical methods in natural language processing-Volume 10*, pages 1–8. Association for Computational Linguistics, 2002.

Michael Collins and Brian Roark. Incremental parsing with the perceptron algorithm. In *Proceedings of the 42nd Annual Meeting on Association for Computational Linguistics*, page 111. Association for Computational Linguistics, 2004.

Michael Collins, Amir Globerson, Terry Koo, Xavier Carreras, and Peter L Bartlett. Exponentiated gradient algorithms for conditional random fields and max-margin markov networks. *The Journal of Machine Learning Research*, 9:1775–1822, 2008.

Koby Crammer, Ryan McDonald, and Fernando Pereira. Scalable large-margin online learning for structured classification. In *NIPS Workshop on Learning With Structured Outputs*, 2005.

Jason M Eisner. Three new probabilistic models for dependency parsing: An exploration. In *Proceedings of the 16th conference on Computational linguistics-Volume 1*, pages 340–345. Association for Computational Linguistics, 1996.

Yoav Freund and Robert E. Schapire. Large margin classification using the perceptron algorithm. *Mach. Learn.*, 37(3):277–296, December 1999. ISSN 0885-6125. doi: 10.1023/A:1007662407062. URL http://dx.doi.org/10.1023/A:1007662407062.

Liang Huang, Suphan Fayong, and Yang Guo. Structured perceptron with inexact search. In *Proceedings of the 2012 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pages 142–151. Association for Computational Linguistics, 2012.

Terry Koo and Michael Collins. Efficient third-order dependency parsers. In *Proceedings of the 48th Annual Meeting of the Association for Computational Linguistics*, pages 1–11. Association for Computational Linguistics, 2010.

Terry Koo, Xavier Carreras, and Michael Collins. Simple semi-supervised dependency parsing. In *Proceedings of ACL-08: HLT*, pages 595–603, 2008.

References II

Sanjiv Kumar and Martial Hebert. Man-made structure detection in natural images using a causal multiscale random field. In *2003 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2003), 16-22 June 2003, Madison, WI, USA*, pages 119–126, 2003. doi: 10.1109/CVPR.2003.1211345. URL https://doi.org/10.1109/CVPR.2003.1211345.

John D. Lafferty, Andrew McCallum, and Fernando C. N. Pereira. Conditional random fields: Probabilistic models for segmenting and labeling sequence data. In *Proceedings of the Eighteenth International Conference on Machine Learning*, ICML '01, pages 282–289, San Francisco, CA, USA, 2001. Morgan Kaufmann Publishers Inc. ISBN 1-55860-778-1. URL http://dl.acm.org/citation.cfm?id=645530.655813.

Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11): 2278–2324, 1998.

Andrew McCallum, Dayne Freitag, and Fernando CN Pereira. Maximum entropy markov models for information extraction and segmentation. In *Icml*, volume 17, pages 591–598, 2000.

Ryan McDonald and Fernando Pereira. Online learning of approximate dependency parsing algorithms. In *11th Conference of the European Chapter of the Association for Computational Linguistics*, 2006.

Ryan McDonald, Fernando Pereira, Kiril Ribarov, and Jan Hajič. Non-projective dependency parsing using spanning tree algorithms. In *Proceedings of the conference on Human Language Technology and Empirical Methods in Natural Language Processing*, pages 523–530. Association for Computational Linguistics, 2005.

Joakim Nivre. Algorithms for deterministic incremental dependency parsing. *Computational Linguistics*, 34(4):513–553, December 2008. ISSN 0891-2017.

Shai Shalev-Shwartz, Yoram Singer, Nathan Srebro, and Andrew Cotter. Pegasos: Primal estimated sub-gradient solver for svm. *Mathematical programming*, 127(1):3–30, 2011.

Ioannis Tsochantaridis, Thorsten Joachims, Thomas Hofmann, and Yasemin Altun. Large margin methods for structured and interdependent output variables. *Journal of machine learning research*, 6(Sep):1453–1484, 2005.

Yue Zhang and Stephen Clark. Syntactic processing using the generalized perceptron and beam search. *Computational linguistics*, 37(1):105–151, 2011.